CP asymmetries and branching ratios of $B \to K\pi$ in supersymmetric models

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ABSTRACT: We analyze the supersymmetric contributions to the direct and mixing CP asymmetries and also to the branching ratios of the $B \to K\pi$ decays in a model independent way. We consider both gluino and chargino exchanges and emphasize that a large gluino contribution is essential for saturating the direct and mixing CP asymmetries. We also find that combined contributions from the penguin diagrams with chargino and gluino in the loop could lead to a possible solution for the branching ratios puzzle and account for the results of R_c and R_n within $b \to s\gamma$ constraints. When all relevant constraints are satisfied, our result indicates that supersymmetry favors lower values of R_c. Finally we study the correlations between the mixing CP asymmetry $S_{K^0\pi^0}$ and mixing CP asymmetries of the processes $B \to \phi K$ and $B \to \eta' K$. We show that it is quite possible for gluino exchanges to accommodate the results of that observables

Contents

1. Introduction

Recently the BaBar and Belle collaborations have measured the CP averaged branching ratios and the CP violating asymmetries of $B \to K\pi$ decays [1–3]. These results, in addition to those from the $B \to \phi K$ and $B \to \eta' K$, offer an interesting avenue to understand the CP violation and flavor mixing of the quark sector in the Standard Model (SM).

In the SM, all CP violating observables should be explained by one complex phase δ_{CKM} in the quark mixing matrix. The effect of this phase has been observed in kaon system. In order to account for the observed CP violation in this sector, δ_{CKM} has to be of order one. With such a large value of δ_{CKM}, the experimental results of the CP asymmetry of $B \to J/\psi K_S$ are consistent with the SM. However, the experimental measurements of the CP asymmetries of $B \to \phi K$, $B \to \eta' K$ and $B \to K\pi$ decays exhibit a possible discrepancy from the SM predictions. Furthermore, it is well known that the strength of the SM CP violation can not generate the observed size of the baryon asymmetry of the universe, and new source of CP violation beyond the δ_{CKM} is needed.

Decay channel	BR $\times 10^6$	A_{CP}	S_f
$\bar{K}^0\pi^-$	24.1 ± 1.3	-0.02 ± 0.034	$-$
$K^-\pi^0$	12.1 ± 0.8	0.04 ± 0.04	$-$
$K^-\pi^+$	18.2 ± 0.8	-0.113 ± 0.019	$-$
$\bar{K}^0\pi^0$	11.5 ± 1.0	-0.09 ± 0.14	0.34 ± 0.28

Table 1: The current experimental results for the CP averaged branching ratios and CP asymmetries of $B \to K\pi$ decays.

In supersymmetric extensions of the SM, there are additional sources of CP violating phases and flavor mixings. It is also established that the SUSY flavor dependent (off-diagonal) phases could be free from the stringent electric dipole moment (EDM) constraints [4]. These phases can easily provide an explanation for the above mentioned anomalies in the CP asymmetries of $B \to \phi K$ and $B \to \eta' K$ [5–7]. We aim in this article to prove that in this class of SUSY models, it is also possible to accommodate the recent experimental results of $B \to K\pi$ CP asymmetries and branching ratios.

The latest experimental measurements for the four branching ratios and the four CP asymmetries of $B \to K\pi$ [1] are given in Table 1. As can be seen from this table, the measured value of the direct CP violation in $\bar{B}^0 \to K^-\pi^+$ is $A^{CP}_{K^-\pi^+} = -0.113 \pm 0.019$ which corresponds to a 4.2σ deviation from zero. While the measured value of $A^{CP}_{K^+\pi^0}$, which may also exhibit a large asymmetry, is quite small. As we will see in the next section, it is very difficult in the SM to get such different values for the CP asymmetries.

Also from these results, one finds that the ratios R_c, R_n and R of $B \to K\pi$ decays are given by

$$R_c = 2\left[\frac{BR(B^+ \to K^+\pi^0) + BR(B^- \to K^-\pi^0)}{BR(B^+ \to K^0\pi^+) + BR(B^- \to \bar{K}^0\pi^-)}\right] = 1.00 \pm 0.08, \qquad (1.1)$$

$$R_n = \frac{1}{2}\left[\frac{BR(B^0 \to K^+\pi^-) + BR(\bar{B}^0 \to K^-\pi^+)}{BR(B^0 \to K^0\pi^0) + BR(\bar{B}^0 \to \bar{K}^0\pi^0)}\right] = 0.79 \pm 0.08, \qquad (1.2)$$

$$R = \left[\frac{BR(B^0 \to K^+\pi^-) + BR(\bar{B}^0 \to K^-\pi^+)}{BR(B^+ \to K^0\pi^+) + BR(B^- \to \bar{K}^0\pi^-)}\right]\frac{\tau^+_B}{\tau_{B^0}} = 0.82 \pm 0.06. \qquad (1.3)$$

In the SM the R_c and R_n ratios are approximately equal, however, the experimental results in Eqs.(1.1,1.2) indicate to 2.4σ deviation from the SM prediction. On the other hand the quantity R is consistent with the SM value. Here $\tau^+_B/\tau_{B^0} = 1.089 \pm 0.017$. These inconsistencies between the $A^{CP}_{K\pi}$ and the $R_c - R_n$ measurements and the SM results are known as $K\pi$ puzzles.

These puzzles have created a lot of interest and several research work have been done to explain the experimental data [8, 9]. It is tempting to conclude that any new physics contributes to $B \to K\pi$ should include a large electroweak penguin in order to explain these discrepancies. In SUSY models, the Z penguin diagrams with chargino exchange in the loop contribute to the electroweak penguin significantly for a light right handed stop mass. Also the subdominant color suppressed electroweak penguin can be enhanced by the electromagnetic penguin with chargino in the loop. Therefore, the supersymmetric extension of the SM is an interesting candidate for explaining the $K\pi$ puzzles.

It is worth mentioning that also new precision determinations of the branching ratios and CP asymmetries of $B \to \pi\pi$ have been recently reported [2, 3]. However, the SUSY contributions to $B \to \pi\pi$, at the quark level, is due to the loop correction for the process $b \to dq\bar{q}$, while the SUSY contribution to $B \to K\pi$ is due to the process $b \to sq\bar{q}$. Therefore, these two contributions are in general independent and SUSY could have significant effect to $B \to K\pi$ and accommodates the new result, while its contribution to $B \to \pi\pi$ remains small. Thus we will focus here only on SUSY contributions to $B \to K\pi$.

In this paper, we perform a detailed analysis of SUSY contributions to the CP asymmetries and the branching ratios of $B \to K\pi$ processes. We emphasize that chargino contribution has the potential to enhance the electroweak penguins and provides a natural solution to the above discrepancies. However, this contribution alone is not large enough to accommodate the experimental results and to solve the $K\pi$ puzzles. We argue that the gluino contribution plays an essential rule in explaining the recent measurements, specially the results of the CP asymmetries. Recall that other supersymmetric contributions like the neutralino and charged Higgs are generally small and can be neglected. The charged Higgs contributions are only relevant at a very large $\tan\beta$ and small charged Higgs mass. Therefore, we are going to concentrate on the chargino and gluino contributions only.

The paper is organized as follows. In section 2 we study the CP asymmetries and the branching ratios of $B \to K\pi$ in the SM. We show that within the SM the $K\pi$ puzzles can not be resolved. In section 3 we analyze the supersymmetric contributions, namely the gluino and chargino contributions, to $B \to K\pi$. We show that a small value of the right-handed stop mass and a large mixing between the second and the third generation in the up-squark mass matrix are required to enhance the chargino Z-penguin. Also a large value of $\tan\beta$ is necessary to increase the effect of the chargino electromagnetic penguin.

Section 4 is devoted to the constraints on SUSY flavor structure from the branch-

ing ratio of $b \to s\gamma$. New upper bounds on the relevant mass insertions are derived in case of dominant gluino or chargino contribution. A correlation between the mass insertions $(\delta_{LR}^d)_{23}$ and $(\delta_{LL}^u)_{32}$ is obtained when both gluino and chargino exchanges are assumed to contribute significantly. In section 5 the SUSY resolution for the $R_c - R_n$ puzzle is considered. We show that it is very difficult to explain this puzzle with a single mass insertion contribution. We emphasize that with simultaneous contributions from gluino and chargino one may be able to explain these discrepancies.

In section 6 we focus on the CP asymmetries in $B \to K\pi$ processes. We show that with a large gluino contribution it is quite natural to account for the recent experimental results of direct CP asymmetries. The SUSY contributions to the mixing CP asymmetry of $B^0 \to K^0\pi^0$ is also discussed. Finally, section 7 contains our main conclusions.

2. $B \to K\pi$ in the Standard Model

In this section we analyze the SM predictions for the CP asymmetries and the branching ratios of $B \to K\pi$ decays. The effective Hamiltonian of $\Delta B = 1$ transition governing these processes can be expressed as

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + h.c., \quad (2.1)$$

where $\lambda_p = V_{pb} V_{ps}^*$ and C_i are the Wilson coefficients and Q_i are the relevant local operators which can be found in Ref.[10]. Within the SM, the $b \to s$ transition can be generated through exchange of W-boson. The Wilson coefficients which describes such a transition can be found in Ref.[10]

The calculation of the decay amplitudes of $B \to K\pi$ involves the evaluation of the hadronic matrix elements of the above operators in the effective Hamiltonian, which is the most uncertain part of this calculation. Adopting the QCD factorization [11], the matrix elements of the effective weak Hamiltonian can be written as

$$\langle \pi K | H_{eff} | \bar{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \pi K | \left(\mathcal{T}_p + \mathcal{T}_p^{ann} \right) | \bar{B} \rangle, \quad (2.2)$$

where

$$\langle \pi K | \mathcal{T}_p | \bar{B} \rangle = \sum_{i=1}^{10} a_i(\pi K) \langle \pi K | Q_i | \bar{B} \rangle_F, \quad (2.3)$$

and

$$\langle \pi K | \mathcal{T}_p^{ann} | \bar{B} \rangle = f_B f_K f_\pi \sum_{i=1}^{10} b_i(\pi K). \quad (2.4)$$

The term $\mathcal{T}_P$ arises from the vertex corrections, penguin corrections and hard spectator scattering contributions which are involved in the parameters $a_i(\pi K)$. The $\langle \pi K | Q_i | \bar{B} \rangle_F$ are the factorizable matrix elements, i.e. if any operator $Q = j_1 \otimes j_2$, then $\langle \pi K | Q_i | \bar{B} \rangle_F = \langle \pi | j_1 | \bar{B} \rangle \langle K | j_2 | 0 \rangle$ or $\langle K | j_1 | \bar{B} \rangle \langle \pi | j_2 | 0 \rangle$. The other term $\mathcal{T}_p^{ann}$ includes the weak annihilation contributions which are absorbed in the parameters $b_i(\pi K)$. Following the notation of Ref.[11] we write the decay amplitude of $B \to K\pi$ as:

$$A_{B^- \to \pi^- \overline{K}^0} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} \left[\delta_{pu}\beta_2 + \alpha_4^p - \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p + \beta_{3,EW}^p \right] \tag{2.5}$$

$$\sqrt{2} A_{B^- \to \pi^0 K^-} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} \left[\delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p + \beta_{3,EW}^p \right]$$

$$+ \sum_{p=u,c} \lambda_p A_{\overline{K}\pi} \left[\delta_{pu}\alpha_2 + \frac{3}{2}\alpha_{3,EW}^p \right] \tag{2.6}$$

$$A_{\overline{B}^0 \to \pi^+ K^-} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} \left[\delta_{pu}\alpha_1 + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p \right] \tag{2.7}$$

$$\sqrt{2} A_{\overline{B}^0 \to \pi^0 \overline{K}^0} = \sum_{p=u,c} \lambda_p A_{\pi \overline{K}} \left[-\alpha_4^p + \frac{1}{2}\alpha_{4,EW}^p - \beta_3^p + \frac{1}{2}\beta_{3,EW}^p \right]$$

$$+ \sum_{p=u,c} \lambda_p A_{\overline{K}\pi} \left[\delta_{pu}\alpha_2 + \frac{3}{2}\alpha_{3,EW}^p \right]. \tag{2.8}$$

Here the coefficients of the flavor operators $\alpha_i^p(\pi K)$ and $\beta_i^p(\pi K)$ are given in terms of the coefficients $a_i^p(\pi K)$ and $b_i^p(\pi K)$ respectively [11]. The parameter $A_{\pi \bar{K}}$ $(A_{\bar{K}\pi})$ is given by $i\frac{G_F}{\sqrt{2}}m_B^2 F_0^{B \to \pi(K)} f_{K(\pi)}$. Note that the parameters b_i of the weak annihilation and hard scattering contributions contain infrared divergence which are usually parameterized as

$$X_{A,H} \equiv \left(1 + \rho_{A,H}e^{i\phi_{A,H}}\right) \ln\left(\frac{m_B}{\Lambda_h}\right), \tag{2.9}$$

where $\rho_{A,H}$ are free parameters to be of order one, $\phi_{A,H} \in [0, 2\pi]$, and $\Lambda_h = 0.5$. As discussed in Ref.[5], the experimental measurements of the branching ratios impose upper bound on the parameter ρ_A. If one does not assume fine tuning between the parameters ρ and ϕ, the typical upper bound on ρ_A is of order of $\rho_A \lesssim 2$.

Fixing the experimental and the SM parameters to their center values, one can determine the explicit dependence of the decay amplitudes of the $B \to K\pi$ on the corresponding Wilson coefficients. For instance, with $\gamma = \pi/3$, and $\rho_{A,H}$ and $\phi_{A,H}$ are of order one, the decay amplitude of $\bar{B}^0 \to K^-\pi^+$ is given by

$$A_{\bar{B}^0 \to \pi^+ K^-} \times 10^8 \simeq (1.05 - 0.02\ i)C_1 + (0.24 + 0.07\ i)C_2 + (3.1 + 14.5\ i)C_3$$

$$+ (4.9 + 37.7\ i)C_4 - (2.9 - 13.1\ i)C_5 + (5.5 - 43.7\ i)C_6$$

$$+ (1.7 + 10.4\ i)C_7 + (5.8 + 36.5\ i)C_8 + (2.8 + 12.7\ i)C_9 \qquad (2.10)$$
$$+ (0.6 + 35.5\ i)C_{10} - (0.0006 + 0.04\ i)C_{7\gamma}^{eff} - (0.04 + 2.5\ i)C_{8g}^{eff}.$$

Similar expression can be obtained for $\bar{B}^0 \to K^0\pi^0$:

$$A_{\bar{B}^0 \to \pi^0 K^0} \times 10^8 \simeq (-0.14 + 0.3\ i)C_1 + (0.4 + 0.2\ i)C_2 - (2.2 + 10.5\ i)C_3$$
$$- (3.5 + 26.7\ i)C_4 + (2.1 - 9.3\ i)C_5 + (3.9 - 30.9\ i)C_6$$
$$- (1.7 + 37.02\ i)C_7 - (1.9 + 1.7\ i)C_8 + (1.3 + 46.6\ i)C_9 \qquad (2.11)$$
$$+ (2.2 + 28.8\ i)C_{10} - (0.0002 + 0.01\ i)C_{7\gamma}^{eff} + (0.03 + 1.8\ i)C_{8g}^{eff}.$$

The amplitude of $B^- \to K^-\pi^0$ can be written as

$$A_{B^- \to \pi^0 K^-} \times 10^8 \simeq (0.9 + 0.06\ i)C_1 + (0.6 + 0.3\ i)C_2 + (2.2 + 10.5\ i)C_3$$
$$+ (3.5 + 26.7\ i)C_4 - (2.1 - 9.3\ i)C_5 - (3.9 - 30.9\ i)C_6$$
$$- (2.7 + 32.4\ i)C_7 - (5.4 - 17.8\ i)C_8 + (1.9 + 51.6\ i)C_9 \qquad (2.12)$$
$$+ (2.4 + 42.8\ i)C_{10} - (0.0004 + 0.03\ i)C_{7\gamma}^{eff} - (0.03 + 1.8\ i)C_{8g}^{eff}.$$

Finally, the amplitude of $B^- \to K^0\pi^-$ is given by

$$A_{B^- \to \pi^- K^0} \times 10^8 \simeq (0.4 - 0.4\ i)C_1 - (0.00004 - 0.02\ i)C_2 + (3.1 + 14.9\ i)C_3$$
$$+ (4.9 + 37.7\ i)C_4 - (2.9 - 13.1\ i)C_5 + (5.5 - 43.7\ i)C_6$$
$$- (3.2 + 3.8\ i)C_7 - (10.8 + 13.8\ i)C_8 - (1.9 + 5.7\ i)C_9 \qquad (2.13)$$
$$- (0.3 + 19.7\ i)C_{10} + (0.0003 + 0.02\ i)C_{7\gamma}^{eff} - (0.04 + 2.5\ i)C_{8g}^{eff},$$

where $C_{7\gamma}^{eff} = C_{7\gamma} - \frac{1}{3}C_5 - C_6$ and $C_{8g}^{eff} = C_{8g} + C_5$. The SM contributions to the Wilson coefficients of $b \to s$ transition, which are the relevant ones for $B \to K\pi$, are given by

$$\mathbf{C}_1^{SM} \simeq 1.077, \quad \mathbf{C}_2^{SM} \simeq -0.175, \quad \mathbf{C}_3^{SM} \simeq 0.012, \quad \mathbf{C}_4^{SM} \simeq -0.33, \quad \mathbf{C}_5^{SM} \simeq 0.0095,$$
$$\mathbf{C}_6^{SM} \simeq -0.039, \quad \mathbf{C}_7^{SM} \simeq 0.0001, \quad \mathbf{C}_8^{SM} \simeq 0.0004, \quad \mathbf{C}_9^{SM} \simeq -0.01, \quad \mathbf{C}_{10}^{SM} \simeq 0.0019,$$
$$\mathbf{C}_{7\gamma}^{SM} \simeq -0.315, \quad \mathbf{C}_{8g}^{SM} \simeq -0.149. \qquad (2.14)$$

From these values, it is clear that within the SM, the dominant contribution to the $B \to K\pi$ decay amplitudes comes from the QCD penguin operator Q_4. However the QCD penguin preserves the isospin. Therefore, this contribution is the same for all the decay modes. Isospin violating contributions to the decay amplitudes arise from the current-current operators Q_1^u and Q_2^u which are called 'tree' contribution and from the electroweak penguins which are suppressed by a power α/α_s. As can be seen from the coefficients of C_{7-10} in Eqs.(2.10-2.13), the electroweak penguin

contributions to the amplitudes of $B \to K\pi$ could be in general sizable and non universal. However, due to the small values of the corresponding Wilson coefficients in the SM (2.14), these contributions are quite suppressed.

Note also that the Q_1 contribution to $A_{\bar{B}^0 \to \pi^+ K^-}$ and $A_{B^- \to \pi^0 K^-}$ is one order of magnitude larger than its contribution to the other two decay amplitudes. Therefore, in the SM, the amplitudes $A_{\bar{B}^0 \to \pi^+ K^-}$ and $A_{B^- \to \pi^0 K^-}$ can be approximated as function of C_1 and C_4, while the amplitudes $A_{\bar{B}^0 \to \pi^0 K^0}$ and $A_{B^- \to \pi^- K^0}$ are approximately given in terms of C_4 only. It is worth noting that the difference between the coefficients of C_4 in the amplitudes $A_{\bar{B}^0 \to \pi^+ K^-}$ and $A_{B^- \to \pi^0 K^-}$ is just due to the factor of $\sqrt{2}$ in Eq.(2.6), which is the same difference between the corresponding coefficients in $A_{\bar{B}^- \to \pi^- K^0}$ and $-A_{B^0 \to \pi^0 K^0}$.

We are now in a position to determine the SM results for the CP asymmetries and the CP average branching ratios of $B \to K\pi$ decays within the framework of the QCD factorization approximation. The direct CP violation may arise in the decay $B \to K\pi$ from the interference between the tree and penguin diagrams. The direct CP asymmetry of $B^0 \to K^-\pi^+$ decay $A^{CP}_{K^-\pi^+}$ is defined as

$$A^{CP}_{K^-\pi^+} = \frac{|A(B^0 \to K^-\pi^+)|^2 - |A(\bar{B}^0 \to K^+\pi^-)|^2}{|A(B^0 \to K^-\pi^+)|^2 + |A(\bar{B}^0 \to K^+\pi^-)|^2}, \tag{2.15}$$

and similar expressions for the asymmetries $A^{CP}_{\bar{K}^0\pi^-}$, $A^{CP}_{K^-\pi^0}$ and $A^{CP}_{\bar{K}^0\pi^0}$. Also the branching ratio can be written in terms of the corresponding decay amplitude as

$$BR(B \to K\pi) = \frac{1}{8\pi} \frac{|P|}{M_B^2} |A(B \to K\pi)|^2 \frac{1}{\Gamma_{tot}}, \tag{2.16}$$

where

$$|P| = \frac{[(M_B^2 - (m_K + m_\pi)^2)(M_B^2 - (m_K - m_\pi)^2)]^2}{2M_B} \tag{2.17}$$

The SM results are summarized in Tables 2 and 3. In Table 2, we present the predictions for the branching ratios of the four decay modes of $B \to \pi K$. We assume that $\gamma = \pi/3$ and consider some representative values of $\rho_{A,H}$ and $\phi_{A,H}$ to check the corresponding uncertainty. Namely, $\rho_{A,H} = 0, 1, 3$ and $\phi_{A,H} = \mathcal{O}(1)$ are considered. From these results, one can see that for $\rho_{A,H} \in [0,1]$ the SM predicted values for the branching ratios of $B \to K\pi$ are less sensitive to the hadronic parameters. Larger values of $\rho_{A,H}$ enhance the branching ratios and eventually they exceed the experimental limits presented in table 1 for $\rho > 2$. It is also remarkable that the SM results for the $BR(B^- \to \bar{K}^0\pi^-)$ and $BR(\bar{B}^0 \to \bar{K}^0\pi^0)$ are larger than the experiment measurements, while the results for $BR(B^- \to K^-\pi^0)$ and $BR(\bar{B}^0 \to K^-\pi^+)$ are consistent with their experimental values. This discrepancy does not

Branching ratio	$\rho_{A,H} = 0$	$\rho_{A,H} = 1$ & $\phi_{A,H} \sim 1$	$\rho_{A,H} = 3$ & $\phi_{A,H} \sim 1$
$BR_{\bar{K}^0\pi^-} \times 10^6$	31.06	33.35	43.92
$BR_{K^-\pi^0} \times 10^6$	17.31	18.45	23.36
$BR_{K^-\pi^+} \times 10^6$	25.87	27.98	39.55
$BR_{\bar{K}^0\pi^0} \times 10^6$	11.41	12.47	18.66
R_n	1.13	1.12	1.059
R_c	1.11	1.106	1.063
R_n	0.83	0.838	0.9

Table 2: The SM predictions for the branching ratios of the four decay modes of $B \to K\pi$ with $\gamma = \pi/3$

seem to be resolved in the SM, even if we consider large hadronic uncertainties. The parameters R_c and R_n, defined in Eqs.(1.1,1.2) as the ratio of the CP average branching ratios of $B \to K\pi$ exhibit this deviation from the SM prediction in a clear way. The results in Table 2 show that in the SM $R_c \simeq R_n > 1$. However, the recent experimental measurements reported in Table 1, implies that $R_c \sim 1$ and $R_n < 1$. It is very difficult to have this situation within the SM. As emphasized above, in the SM the amplitudes of $B \to K\pi$ can be approximately written as

$$A_{\bar{B}^0 \to \pi^+ K^-} \simeq (a_1 + b_1 \ i) \ C_1 + (a_2 + b_2 \ i) \ C_4, \tag{2.18}$$

$$A_{\bar{B}^0 \to \pi^0 K^0} \simeq -\frac{1}{\sqrt{2}}(a_2 + b_2 \ i) \ C_4, \tag{2.19}$$

$$A_{B^- \to \pi^0 K^-} \simeq \frac{1}{\sqrt{2}}(a_1 + b_1 \ i) \ C_1 + \frac{1}{\sqrt{2}}(a_2 + b_2 \ i) \ C_4 \tag{2.20}$$

$$A_{B^- \to \pi^- K^0} \simeq (a_2 + b_2 \ i) \ C_4. \tag{2.21}$$

Thus, the parameters R_c and R_n are given by

$$R_c = R_n = \frac{|(a_1 + b_1 \ i) \ C_1 + (a_2 + b_2 \ i) \ C_4|^2}{|(a_2 + b_2 \ i) \ C_4|^2} \gtrsim 1 \tag{2.22}$$

which is consistent with the result given in Table 2, using the full set of the Wilson coefficients.

Now we turn to the SM predictions for the CP asymmetries of $B \to K\pi$. Let us start by considering the approximation that the decay amplitudes for $B^- \to K^0\pi^-$ and $\bar{B}^0 \to K^0\pi^0$ are dominated by the pure gluon penguin operator Q_4 while the amplitudes for $B^- \to K^-\pi^0$ and $\bar{B}^0 \to K^-\pi^+$ are given by Q_4 and also by the tree contribution of the current-current operator Q_1. In this case, the following results are expected: The direct CP asymmetries $A^{CP}_{K^0\pi^-}$ and $A^{CP}_{K^0\pi^0}$ should be very tiny (equal

CP asymmetry	$\rho_{A,H} = 0$	$\rho_{A,H} = 1$ & $\phi_{A,H} \sim 1(-1)$	$\rho_{A,H} = 3$ & $\phi_{A,H} \sim 1(-1)$
$A^{CP}_{\bar{K}^0\pi^-}$	0.007	0.0086 (0.005)	0.0078 (0.001)
$A^{CP}_{K^-\pi^0}$	0.029	0.063 (-0.006)	0.185 (-0.15)
$A^{CP}_{K^-\pi^+}$	0.0044	0.057 (-0.049)	0.194 (-0.19)
$A^{CP}_{\bar{K}^0\pi^0}$	-0.02	-0.013 (-0.025)	-0.019 (-0.002)

Table 3: The SM predictions for the direct CP asymmetries of the four decay modes of $B \to \pi K$ with $\gamma = \pi/3$

zero in the exact limit of this approximation). The direct CP asymmetries $A^{CP}_{K^0\pi^0}$ and $A^{CP}_{K^-\pi^+}$ should be of the same order and larger than the other two asymmetries.

The SM results of the CP asymmetries for the different decay modes, including the effect of all local operators Q_i, are given in Table 3. As in the case of the branching ratios, we assume that $\gamma = \pi/3$, and $\rho_{A,H} = 0, 1, 3$. Respect to the strong phases $\phi_{A,H}$, we take it to be of order one as before. Due to the sensitivity of the CP asymmetry on their sign, we consider both cases of $\phi_{A,H} = \mathcal{O}(\pm 1)$. Few comments on the results of the direct CP asymmetries given in Table 2 are in order:

1. The CP asymmetries $A^{CP}_{K^-\pi^0}$ and $A^{CP}_{K^-\pi^+}$ are sensitive to the sign ϕ_A (note that ϕ_H is irrelevant for these processes). On the contrary, the CP asymmetries $A^{CP}_{K^0\pi^-}$ and $A^{CP}_{K^0\pi^0}$ are insensitive to this sign.

2. As expected, the results of the CP asymmetries $A^{CP}_{K^0\pi^-}$ and $A^{CP}_{K^0\pi^0}$ are very small even with large values of ρ_A.

3. The value of $A^{CP}_{K^-\pi^0}$ and $A^{CP}_{K^-\pi^+}$ can be enhanced by considering large value of ρ_A and one gets values for $A^{CP}_{K^-\pi^+}$ of order the experimental result given in Table 1. However, it is very important to note that in this case, the CP asymmetry $A^{CP}_{K^-\pi^0}$ is also enhanced in the same way and it becomes one order of magnitude larger than its experimental value.

While a confirmation with more accurate experimental data is necessary, the above results of the branching ratio and the direct CP asymmetries of $B \to \pi K$ show that within the SM the current experimental measurements listed in Table 1 do not seems to be accommodated even if one considers large hadronic uncertainties. It is worth stressing that the QCD correction would not play an essential role in solving this $K\pi$ puzzles. Furthermore, since we are interested here in the ratio of the amplitudes, many of the theoretical uncertainties cancel. So it can not be the source of these discrepancies.

Another useful way of parameterizing the decay amplitudes can be obtained by factorizing the dominant penguin amplitude P, where P is defined as [12]

$$Pe^{i\delta_P} = \alpha_4^c - \frac{1}{2}\alpha_{4,EW}^c + \beta_3^c + \beta_{3,EW}^c. \tag{2.23}$$

In this case, one can write the above expressions for the decay amplitude as follows:

$$\begin{aligned}
A_{B^- \to \pi^- \overline{K}^0} &= \lambda_c A_{\pi\overline{K}} P \left[1 + r_A e^{i\delta_A} e^{-i\gamma}\right], \\
\sqrt{2} A_{B^- \to \pi^0 K^-} &= \lambda_c A_{\pi\overline{K}} P \left[1 + \left(r_A e^{i\delta_A} + r_C e^{i\delta}\right) e^{-i\gamma} + r_{EW} e^{i\delta_{EW}}\right], \\
A_{\overline{B}^0 \to \pi^+ K^-} &= \lambda_c A_{\pi\overline{K}} P \left[1 + \left(r_A e^{i\delta_A} + r_T e^{i\delta_T}\right) e^{-i\gamma} + r_{EW}^C e^{i\delta_{EW}^C}\right], \\
-\sqrt{2} A_{\overline{B}^0 \to \pi^0 \overline{K}^0} &= \lambda_c A_{\pi\overline{K}} P \left[1 + \left(r_A e^{i\delta_A} + r_T e^{i\delta_T} - r_C e^{i\delta_C}\right) e^{-i\gamma} + r_{EW}^C e^{i\delta_{EW}^C} - r_{EW} e^{i\delta_{EW}}\right],
\end{aligned} \tag{2.24}$$

where

$$r_A e^{i\delta_A} = \epsilon_{KM} \left[\beta_2 + \alpha_4^u - \frac{1}{2}\alpha_{4,EW}^u + \beta_3^u + \beta_{3,EW}^u\right]/P, \tag{2.25}$$

$$r_T e^{i\delta_T} = \epsilon_{KM} \left[\alpha_1 + \frac{3}{2}\alpha_{4,EW}^u - \frac{3}{2}\beta_{3,EW}^u - \beta_2\right]/P, \tag{2.26}$$

$$r_C e^{i\delta_P} = \epsilon_{KM} \left[\alpha_1 + R_{K\pi}\alpha_2 + \frac{3}{2}(R_{K\pi}\alpha_{3,EW}^u + \alpha_{4,EW}^u)\right]/P, \tag{2.27}$$

$$r_{EW} e^{i\delta_{EW}} = \left[\frac{3}{2}(R_{K\pi}\alpha_{3,EW}^c + \alpha_{4,EW}^c)\right]/P, \tag{2.28}$$

$$r_{EW}^C e^{i\delta_{EW}^C} = \left[\frac{3}{2}(\alpha_{4,EW}^c - \beta_{3,EW}^c)\right]/P. \tag{2.29}$$

Here we define $\lambda_u/\lambda_c \equiv \epsilon_{KM}e^{-i\gamma}$, $R_{K\pi} = A_{\pi\overline{K}}/A_{\overline{K}\pi}$, and δ_A, δ_T, δ_C, δ_{EW} and δ_{EW}^C as strong interaction phases. The SM contributions within the QCD facorization leads to the following results:

$$\begin{aligned}
(Pe^{i\delta_P})_{SM} &= -0.11e^{0.051i}, & (r_A e^{i\delta_A})_{SM} &= 0.019e^{0.26i}, \\
(r_C e^{i\delta_C})_{SM} &= 0.186e^{2.9i}, & (r_T e^{i\delta_T})_{SM} &= 0.191e^{2.9i}, \\
(r_{EW} e^{i\delta_{EW}})_{SM} &= 0.13e^{-0.2i}, & (r_{EW}^C e^{i\delta_{EW}^C})_{SM} &= 0.012e^{-2.5i}.
\end{aligned} \tag{2.30}$$

As can be seen from this result, within the SM r_A and r_{EW}^C are much smaller than r_C, r_T and r_{EW}, so that they can be easily neglected. In this case, the parameters R_c and R_n can be expressed by the following approximated expressions

$$R_c \simeq 1 + 2r_C \cos\delta_C \cos\gamma + 2r_{EW}\cos\delta_{EW}, \tag{2.31}$$

$$R_n \simeq \frac{1 + 2r_T \cos\delta_T \cos\gamma}{1 + 2r_T \cos\delta_T \cos\gamma - 2r_C \cos\delta_C \cos\gamma - 2r_{EW}\cos\delta_{EW}}, \tag{2.32}$$

which confirms our previous conclusion that in the SM $R_n \sim R_c \gtrsim 1$. Explicitly, using the results of Eq.(2.30), one finds that

$$R_c = 1.08(1.45), \qquad R_n = 1.13(1.6), \qquad R = 0.757(0.673) \tag{2.33}$$

for $\gamma = \pi/3(2\pi/3)$, which is quite close to the full result that we obtained in Table 2, with $\rho_A \sim 1$.

Now, we would like to comment on the mixing CP asymmetry of $B \to K\pi$. CP violation in the interference between mixing and decay can be observed as time dependent oscillation of the CP asymmetry. The amplitude of the oscillation in charmonium decay modes provides a theoretical clean determination of the parameter $\sin 2\beta$ of the unitary triangle. The SM predicts the B-decay modes, dominated by a single penguin amplitude such that $B \to \phi K$, $B \to \eta' K$ and $B \to K^0\pi^0$ to have the same time dependent CP asymmetry equal to $\sin 2\beta$. Again this result contradicts the experimental measurement given in Table 1. Note that the latest experimental results on the mixing CP asymmetry of $B \to \phi K_S$ process are given by [2,3]

$$
\begin{aligned}
S_{\phi K_S} &= 0.50 \pm 0.25^{+0.07}_{-0.04} \ \ (\text{BaBar}), \\
&= 0.06 \pm 0.33 \pm 0.09 \ \ (\text{Belle}),
\end{aligned}
\tag{2.34}
$$

where the first errors are statistical and the second systematic. Thus, the average of this CP asymmetry is $S_{\phi K_S} = 0.34 \pm 0.20$. On the other hand, the most recent measured CP asymmetry in the $B^0 \to \eta' K_S$ decay is found by BaBar [2] and Belle [3] collaborations as

$$
\begin{aligned}
S_{\eta' K_S} &= 0.27 \pm 0.14 \pm 0.03 \ \ (\text{BaBar}) \\
&= 0.65 \pm 0.18 \pm 0.04 \ \ (\text{Belle}),
\end{aligned}
\tag{2.35}
$$

with an average $S_{\eta' K_S} = 0.41 \pm 0.11$, which shows a 2.5σ discrepancy from the SM expectation. This difference among $S_{\phi K}$, $S_{\eta' K}$, $S_{K^0\pi^0}$ and $\sin 2\beta$ is also considered as a hint for new physics beyond the SM, in particular for supersymmetry.

3. $B \to K\pi$ in SUSY models

As mentioned in the previous section, due to the asymptotic freedom of QCD, the calculation of the hadronic decay amplitude of $B \to K\pi$ can be factorized by the product of long and short distance contributions. The short distance contributions, including the SUSY effects are contained in the Wilson coefficients C_i.

The SUSY contributions to the $b \to s$ transition could be dominated by the gluino or the chargino intermediated penguin diagrams [5]. It turns out that the

dominant effect in both contributions is given by chromomagnetic penguin (Q_{8g}). However in case of $B \to K\pi$, it was observed that this process is more sensitive to the isospin violating interactions [8, 9], namely the contributions from the electromagnetic penguin ($Q_{7\gamma}$) and photon- and Z-penguins contributions to Q_7 and Q_9. Therefore, in our discussion we will focus only on these contributions, although in our numerical analysis we keep all the contributions of the gluino and chargino.

For the gluino exchange, it turns out that the Z-penguin contributions to $C_{7,9}$ are quite small and can be neglected with respect to the photon-penguin contributions. At the first order in the mass insertion approximation, the gluino contributions to the Wilson coefficients $C_{7\gamma,8g}$, C_7 and C_9 at SUSY scale M_S are given by

$$C_7^{\tilde{g}}(M_S) = C_9(M_S) = \frac{2\alpha_s\alpha}{9\sqrt{2}G_F m_{\tilde{q}}^2} \frac{1}{3}(\delta_{LL}^d)_{23}P_{042}(x,x), \tag{3.1}$$

$$C_{7\gamma}^{\tilde{g}}(M_S) = \frac{8\alpha_s\pi}{9\sqrt{2}G_F m_{\tilde{q}}^2}\left[(\delta_{LL}^d)_{23}M_3(x) + (\delta_{LR}^d)_{23}\frac{m_{\tilde{g}}}{m_b}M_1(x)\right], \tag{3.2}$$

$$C_{8g}^{\tilde{g}}(M_S) = \frac{\alpha_s\pi}{\sqrt{2}G_F m_{\tilde{q}}^2}\left[(\delta_{LL}^d)_{23}\left(\frac{1}{3}M_3(x) + 3M_4(x)\right)\right.$$
$$\left. + (\delta_{LR}^d)_{23}\frac{m_{\tilde{g}}}{m_b}\left(\frac{1}{3}M_3(x) + 3M_2(x)\right)\right], \tag{3.3}$$

where $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ and the functions $M_1(x)$, $M_2(x)$ and $P_{ijk}(x,x)$ can be found in Ref.[13, 14]. The coefficients $\tilde{C}_{7\gamma,8g}$ and $\tilde{C}_{7,9}$ are obtained from $C_{7\gamma,8g}$ and $C_{7,9}$ respectively, by the chirality exchange $L \leftrightarrow R$. As can be seen from Eqs.(3.2,3.3), the term proportional to $(\delta_{LR}^d)_{23}$ in the coefficients $C_{7\gamma,8g}$ has a large enhancement factor $m_{\tilde{g}}/m_b$. This enhancement factor is responsible for the dominant gluino effects in B-decays, although this mass insertion is strongly constrained from $b \to s\gamma$. Note also that, since the photon-penguin gives the same contributions to C_7 and C_9, and we neglect the Z-penguin contributions, we have $C_7 = C_9$. Finally, it is clear that the coefficients $C_{7,9}$ is suppressed with respect to $C_{7\gamma,8g}$ by a factor $\alpha/4\pi$ at least.

It is worth mentioning that the mass insertion $(\delta_{LR}^d)_{23}$ can be generated by the mass insertion $(\delta_{LL}^d)_{23}$ as follows [15]

$$(\delta_{LR}^d)_{23} = (\delta_{LL}^d)_{23}\ (\delta_{LR}^d)_{33},$$

where

$$(\delta_{LR}^d)_{33} \sim \frac{m_b(A_b - \mu\tan\beta)}{m_{\tilde{d}}^2} \sim \frac{m_b}{m_{\tilde{d}}^2}\ \tan\beta \sim 10^{-2}\tan\beta.$$

Therefore,

$$(\delta_{LR}^d)_{23} \simeq 10^{-2}\tan\beta(\delta_{LL}^d)_{23}.$$

Hence, for a moderate value of $\tan\beta$ and $(\delta_{LL}^d)_{23} \sim \mathcal{O}(0.1)$, one obtains $(\delta_{LR}^d)_{23}$ of order 10^{-2}, which can easily imply significant contributions for the $S_{\phi K}$ and also account for the different results between $S_{\phi K}$ and $S_{\eta' K}$. Thus in our analysis we define

$$(\delta_{LR}^d)_{23\text{eff}} = (\delta_{LR}^d)_{23} + (\delta_{LL}^d)_{23} \, (\delta_{LR}^d)_{33}. \tag{3.4}$$

It is important to stress that in case of $(\delta_{LR}^d)_{23\text{eff}}$ dominated by double mass insertions, we still call this scenario as LR contribution. This is due to the fact that the main SUSY contribution is still through the C_{8g} which is enhanced by the chirality flipped factor $m_{\tilde{g}}/m_b$. In the literatures [16], this contribution has been considered in analyzing the CP asymmetry of $B \to \phi K$ and it was called as LL contribution, as indication for the large mixing in the squark mass matrix and dominant effect of $(\delta_{LL}^d)_{23}$. However, we prefer to work with the notation LR_{eff} to be able to trace the effective operators that may lead to dominant contributions for different B decay channels.

The dominant chargino contributions are found to be also due to the chromomagnetic penguin, magnetic penguin and Z-penguin diagrams. As emphasized in Ref.[5], these contributions depend on the up sector mass insertion $(\delta_{LL}^u)_{32}$ and $(\delta_{RL}^u)_{32}$ while the LR and RR contributions are suppressed by λ^2 or λ^3, where λ is the Cabibbo mixing. At the first order in the mass insertion approximation, the chargino contributions to the Wilson coefficients are given by [5]

$$C_7^\chi(M_S) = \frac{\alpha}{6\pi}\left(4C_\chi + D_\chi\right), \tag{3.5}$$

$$C_9^\chi(M_S) = \frac{\alpha}{6\pi}\left(4(1 - \frac{1}{\sin^2\theta_W})C_\chi + D_\chi\right), \tag{3.6}$$

$$C_{7\gamma}^\chi = M_\gamma, \tag{3.7}$$

$$C_{8g}^\chi = M_g, \tag{3.8}$$

where the functions $F \equiv C_\chi$ (Z-penguin), D_χ (photon-penguin), M^γ (magnetic-penguin), and M^g (chromomagnetic penguin) are given by [5]

$$F_\chi = \left[(\delta_{LL}^u)_{32} + \lambda(\delta_{LL}^u)_{31}\right]R_F^{LL} + \left[(\delta_{RL}^u)_{32} + \lambda(\delta_{RL}^u)_{31}\right]Y_t \, R_F^{RL}. \tag{3.9}$$

The functions R_F^{LL} and R_F^{RL}, F depend on the SUSY parameters through the chargino masses (m_{χ_i}), squark masses $(\tilde{m})$and the entries of the chargino mass matrix. For the Z and magnetic (chromomagnetic) dipole penguins $R_C^{LL,RL}$ and $R_{M^{\gamma(g)}}^{LL,RL}$ respectively, we have [5]

$$R_C^{LL} = \sum_{i=1,2} |V_{i1}|^2 \, P_C^{(0)}(\bar{x}_i) + \sum_{i,j=1,2}\left[U_{i1}V_{i1}U_{j1}^\star V_{j1}^\star \, P_C^{(2)}(x_i, x_j)\right.$$

$$+ |V_{i1}|^2 |V_{j1}|^2 \left(\frac{1}{8} - P_C^{(1)}(x_i, x_j) \right) \Bigg] ,$$

$$R_C^{RL} = -\frac{1}{2} \sum_{i=1,2} V_{i2}^\star V_{i1} \, P_C^{(0)}(\bar{x}_i, \bar{x}_{it}) - \sum_{i,j=1,2} V_{j2}^\star V_{i1} \left(U_{i1} U_{j1}^\star \, P_C^{(2)}(x_i, x_{it}, x_j, x_{jt}) \right.$$
$$\left. + V_{i1}^\star V_{j1} \, P_C^{(1)}(x_i, x_j) \right) ,$$

$$R_{M^{\gamma,g}}^{LL} = \sum_i |V_{i1}|^2 \, x_{Wi} \, P_{M^{\gamma,g}}^{LL}(x_i) - Y_b \sum_i V_{i1} U_{i2} \, x_{Wi} \, \frac{m_{\chi_i}}{m_b} P_{M_{\gamma,g}}^{LR}(x_i),$$

$$R_{M^{\gamma,g}}^{RL} = - \sum_i V_{i1} V_{i2}^\star \, x_{Wi} \, P_{M_{\gamma,g}}^{LL}(x_i, x_{it}), \qquad (3.10)$$

where Y_b is the Yukawa coupling of bottom quark, $x_{Wi} = m_W^2/m_{\chi_i}^2$, $x_i = m_{\chi_i}^2/\tilde{m}^2$, $\bar{x}_i = \tilde{m}^2/m_{\chi_i}^2$, and $x_{it} = m_{\chi_i}^2/m_{\tilde{t}_R}^2$. The loop functions $P_{M_{\gamma,g}}^{LL(LR)}$ can be found in Ref.[5]. Finally, U and V are the matrices that diagonalize chargino mass matrix.

Notice that the terms in $R_{M_\gamma}^{LL}$ and $R_{M_g}^{LL}$ which are enhanced by m_{χ_i}/m_b in Eq.(3.10) lead to the large effects of chargino contributions to $C_{7\gamma}$ and C_{8g}, respectively. Also the dependence of these terms on Yukawa bottom Y_b enhance the LL contributions in $C_{7\gamma,8g}$ at large $\tan\beta$. In the case of light stop-right, the function R_C^{RL} of the Z-penguin contribution is largely enhanced. In order to understand the impact of the chargino contributions in $B \to K\pi$ process, it is very useful to present the explicit dependence of the Wilson coefficients $C_{7,9,7\gamma,8g}$ in terms of the relevant mass insertions. For gaugino mass $M_2 = 200$ GeV, squark masses $\tilde{m} = 500$ GeV, light stop $\tilde{m}_{\tilde{t}_R} = 150$ GeV, $\mu = 400$ GeV, and $\tan\beta = 10$, we obtain

$$C_7^\chi \simeq 0.000002(\delta_{LL}^u)_{32} - 0.000011(\delta_{RL}^u)_{31} - 0.000046(\delta_{RL}^u)_{32}, \qquad (3.11)$$

$$C_9^\chi \simeq 0.00000039(\delta_{LL}^u)_{32} + 0.000037(\delta_{RL}^u)_{31} + 0.000165(\delta_{RL}^u)_{32}, \qquad (3.12)$$

$$C_{7\gamma}^\chi \simeq -0.011(\delta_{LL}^u)_{31} - 0.05(\delta_{LL}^u)_{32} - 0.00043(\delta_{RL}^u)_{31} - 0.002(\delta_{RL}^u)_{32}, \qquad (3.13)$$

$$C_{8g}^\chi \simeq -0.0032(\delta_{LL}^u)_{31} - 0.0014(\delta_{LL}^u)_{32} - 0.0003(\delta_{RL}^u)_{31} - 0.0012(\delta_{RL}^u)_{32}. \; (3.14)$$

From these results, it is clear that the Wilson coefficient $C_{7\gamma}^\chi$ seems to give the dominant contribution, specially through the LL mass insertion. However, one should be careful with this contribution since it is also the main contribution to the $b \to s\gamma$, and stringent constraints on $(\delta_{LL}^u)_{32}$ are usually obtained, specially with large $\tan\beta$. Finally, as expected from Eq.(3.10), only LL contributions to $C_{7\gamma}^\chi$ and C_{8g}^χ have strong depend on the value of $\tan\beta$. For instance with $\tan\beta = 40$, these contributions are enhanced with a factor 4, while the result of $C_{7,9}^\chi$ and LR part of $C_{7\gamma}^\chi$ and C_{8g}^χ change from the previous ones by less than 2%.

4. On the constraints from $BR(B \to X_s\gamma)$

In this section we revise the constraints on SUSY flavor structure which arise from the experimental measurements of the branching ratio of the $B \to X_s\gamma$ [17]:

$$2 \times 10^{-4} < BR(b \to s\gamma) < 4.5 \times 10^{-4} \quad \text{(at 95\%C.L.).} \tag{4.1}$$

In supersymmetric models, there are additional contributions to $b \to s\gamma$ decay besides the SM diagrams with W-gauge boson and an up quark in the loop. The SUSY particles running in the loop are: charged Higgs bosons ($H^\pm$) or chargino with up quarks and gluino or neutralino with down squarks. The total amplitude for this decay is sum of all these contributions. As advocated in the introduction, the neutralino contributions are quite small and can be safely neglected. Also the charged Higgs contributions are only relevant at very large $\tan\beta$ and small charged Higgs mass. Therefore, we consider chargino and gluino contributions only to analyze the possible constraints on the mass insertions $(\delta^u_{AB})_{32}$ and $(\delta^d_{AB})_{23}$, where $A \equiv L, R$.

Although the gluino contribution to $b \to s\gamma$ is typically very small in models with minimal flavor structure, it is significantly enhanced in models with non minimal favor structure [18]. In this class of models, both chargino and gluino exchanges give large contribution to the amplitude of $b \to s\gamma$ decay, and hence, they have to be simultaneously considered in analyzing the constraints of the branching ratio $BR(b \to s\gamma)$.

The relevant operators for this process are Q_2, $Q_{7\gamma}$, and Q_{8g}. The contributions of the other operators in Eq.(2.1) can be neglected. The branching ratio $BR(b \to s\gamma)$, conventionally normalized to the semileptonic branching ratio $BR^{exp}(B \to X_c e\nu) = (10.4 \pm 0.4)\%$ [19], is given by [20]

$$BR^{\text{NLO}}(B \to X_s\gamma) = BR^{\text{exp}}(B \to X_c e\nu) \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha_{em}}{\pi g(z)k(z)} \left(1 - \frac{8}{3} \frac{\alpha_s(m_b)}{\pi}\right)$$
$$\times \left(|D|^2 + A\right)(1 + \delta_{np}), \tag{4.2}$$

with

$$D = C_7^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} \left(C_7^{(1)}(\mu) + \sum_{i=1}^{8} C_i^{(0)}(\mu) \left[r_i(z) + \gamma_{i7}^{(0)} \log \frac{m_b}{\mu}\right]\right),$$

$$A = \left(e^{-\alpha_s(\mu)\log\delta(7+2\log\delta)/3\pi} - 1\right)|C_7^{(0)}(\mu)|^2 + \frac{\alpha_s(\mu)}{\pi} \sum_{i \le j=1}^{8} C_i^{(0)}(\mu)C_j^{(0)}(\mu)f_{ij}(\delta),$$

where $z = m_c^2/m_b^2$, μ is the renormalization scale which is chosen of order m_b, and ρ is photon energy resolution. The expressions for $C_i^{(0)}$, $C_i^{(1)}$, and the anomalous

dimension matrix γ, together with the functions $g(z)$, $k(z)$, $r_i(z)$ and $f_{ij}(\delta)$, can be found in Ref. [20]. The term δ_{np} (of order a few percent) includes the non-perturbative $1/m_b$ [21] and $1/m_c$ [22] corrections. From the formula above we obtain the theoretical result for $BR(B \to X_s \gamma)$ in the SM which is given by

$$BR^{\mathrm{NLO}}(B \to X_s \gamma) = (3.29 \pm 0.33) \times 10^{-4} \tag{4.3}$$

where the main theoretical uncertainty comes from uncertainties in the SM input parameters, namely m_t, $\alpha_s(M_Z)$, α_{em}, m_c/m_b, m_b, V_{ij}, and the small residual scale dependence. The central value in Eq.(4.3) corresponds to the following central values for the SM parameters $m_t^{\mathrm{pole}} \simeq m_t^{\overline{\mathrm{MS}}}(m_Z) \simeq 174\,\mathrm{GeV}$, $m_b^{\mathrm{pole}} = 4.8\,\mathrm{GeV}$, $m_c^{\mathrm{pole}} = 1.3\,\mathrm{GeV}$, $\mu = m_b$, $\alpha_s(m_Z) = 0.118$, $\alpha_e^{-1}(m_Z) = 128$, $\sin^2 \theta_W = 0.23$ and a photon energy resolution corresponding to $\rho = 0.9$ is assumed.

The SUSY contributions to the Wilson coefficients $C_{7\gamma}$ and C_{8g} at leading order are given in the previous section. In general, the SUSY effects in $b \to s\gamma$ decay can be parameterized by introducing $R_{7,8}$ and $\tilde{R}_{7,8}$ parameters defined at the electroweak scale as

$$R_{7,8} = \frac{\left(C_{7\gamma,8g} - C_{7\gamma,8g}^{SM}\right)}{C_{7\gamma,8g}^{SM}}, \quad \tilde{R}_{7,8} = \frac{\tilde{C}_{7\gamma,8g}}{C_{7\gamma,8g}^{SM}}, \tag{4.4}$$

where $C_{7\gamma,8g}$ include the total contribution while $C_{7\gamma,8g}^{SM}$ contains only the SM ones. Note that in $\tilde{C}_{7\gamma,8g}$, which are the corresponding Wilson coefficients for $\tilde{Q}_{7\gamma,8g}$ respectively, we have set to zero the SM contribution. Inserting these definitions into the $BR(B \to X_s \gamma)$ formula in Eq.(4.2) yields a general parametrization of the branching ratio [18, 23]

$$
\begin{aligned}
\mathrm{BR}(B \to X_s \gamma) = \mathrm{BR}^{\mathrm{SM}}(B \to X_s \gamma) &\left(1 + 0.681 Re\,(R_7) + 0.116 \left[|R_7|^2 + |\tilde{R}_7|^2\right]\right. \\
&+ 0.083 Re(R_8) + 0.025 \left[Re(R_7 R_8^*) + Re(\tilde{R}_7 \tilde{R}_8^*)\right] \\
&\left.+ 0.0045 \left[|R_8|^2 + |\tilde{R}_8|^2\right]\right) \ .
\end{aligned}
\tag{4.5}
$$

From this parametrization, it is clear that $C_{7\gamma}$ would give the dominant new contribution (beyond the SM one) to the $BR(B \to X_s \gamma)$. Using the allowed experimental range given in Eq.(4.1), one can impose stringent constraints on $C_{7\gamma}$, and hence on the corresponding mass insertions. It is also remarkable that R_7 and $\tilde{R}_7$ have different contributions to the $BR(B \to X_s \gamma)$, therefore, the possible constraints on $C_{7\gamma}$ and hence on the LL and LR mass insertions would be different from the constraints on $\tilde{C}_{7\gamma}$ and hence on the RR and RL mass insertions, unlike what has been assumed in the literatures. Furthermore, since the leading contribution to the branching ratio is due to $Re(R_7)$, the CP violating phase of $C_{7\gamma}$ will play a crucial role in the possible constraints imposed by $BR(B \to X_s \gamma)$.

x	$\lvert(\delta^d_{LR})_{23}\rvert$	$\lvert(\delta^d_{RL})_{23}\rvert$
0.3	(a) 0.0116 (b) 0.0038 (c) 0.0012	0.0038
1	(a) 0.02 (b) 0.006 (c) 0.002	0.006
4	(a) 0.006 (b) 0.015 (c) 0.0045	0.016

Table 4: Upper bounds of $\lvert(\delta^d_{LR(RL)})_{23}\rvert$ from $b \to s\gamma$ decay for $m_{\tilde{q}} = 500$ GeV and $\arg(\delta^{\mathrm{d}}_{\mathrm{LR(RL)}})_{23} = 0$ (a), $\pi/2$ (b), π (c) respectively.

Note that the constraints obtained in Ref. [13], namely $(\delta^d_{LR})_{23} \leq 1.6 \times 10^{-2}$ and $(\delta^d_{LL})_{23}$ is unconstrained are based on the assumption that the gluino amplitude is the dominant contribution to $b \to s\gamma$, even dominant with respect to the SM amplitude. Although this a very acceptable assumption in order to derive a conservative constraints on the relevant mass insertions, it is unrealistic and usually lead to unuseful constraint. The aim of this section is to provide a complete analysis of the $b \to s\gamma$ constraints by including the SM, chargino and gluino contributions.

Let us start first with gluino contribution as the dominant SUSY effect to $b \to s\gamma$ decay. We assume that the average squark mass of order 500 GeV and we consider three representative values for $x = (m_{\tilde{g}}/m_{\tilde{q}})^2 = 0.3, 1$, and 4. We also assume that the SM value for $BR(B \to X_s\gamma)$ is given by 3.29×10^{-4}, which is the central value of the results in Eq.(4.3). In these cases we find that both the mass insertions $\lvert(\delta^d_{LL})_{23}\rvert$ and $\lvert(\delta^d_{RR})_{23}\rvert$ are unconstrained by the branching ratio of $b \to s\gamma$ for any values of their phases. The upper bounds on $\lvert(\delta^d_{LR})_{23}\rvert$ and $\lvert(\delta^d_{RL})_{23}\rvert$ from $b \to s\gamma$ decay are give in Table 4. As can be seen from these results, the limits on $\lvert(\delta^d_{LR})_{23}\rvert$ are quite sensitive to the phase of this mass insertion, unlike the bounds on $\lvert(\delta^d_{RL})_{23}\rvert$. Also, as suggested by Eq.(4.5), the bounds on LR coincides with the ones on RL only if $\arg(\delta^{\mathrm{d}}_{\mathrm{LR(RL)}})_{23} = \pi/2$. Note that in this case $Re(R_7)$ vanishes and the expression of the branching ratio is a symmetric under exchange R_7 and $\tilde{R}_7$.

Now we consider the chargino contribution as the dominant SUSY effect to $b \to s\gamma$ in order to analyze the bounds on the relevant mass insertions in the up squark sector. From the expression of $C^\chi_{7\gamma}$ in Eq.(3.13), which provide the leading contribution to the branching ratio of $b \to s\gamma$, it is clear that one can derive strong

$M_2 \backslash m$	300	500	700	900
150	(a) 0.04	0.065	0.095	0.14
	(b) 0.14	0.24	0.37	0.54
	(c) 0.51	0.85	——	——
250	(a) 0.053	0.075	0.1	0.15
	(b) 0.20	0.28	0.4	0.55
	(c) 0.70	——	——	——
350	(a) 0.07	0.09	0.12	0.16
	(b) 0.26	0.33	0.45	0.6
	(c) 0.92	——	——	——
450	(a) 0.085	0.105	0.14	0.16
	(b) 0.33	0.4	0.5	0.6
	(c) ——	——	——	——

$M_2 \backslash m$	300	500	700	900
150	(a) 0.17	0.28	0.45	0.65
	(b) 0.65	——	——	——
	(c) ——	——	——	——
250	(a) 0.24	0.34	0.48	0.67
	(b) 0.86	——	——	——
	(c) ——	——	——	——
350	(a) 0.32	0.4	0.52	0.73
	(b) ——	——	——	——
	(c) ——	——	——	——
450	(a) 0.45	0.48	0.62	0.8
	(b) ——	——	——	——
	(c) ——	——	——	——

Table 5: Upper bounds of $|(\delta^u_{LL})_{32}|$ (left) and $|(\delta^u_{LL})_{31}|$ (right) from $b \to s\gamma$ decay for $\tan\beta = 10$ and $\mu = 400$ GeV and $\arg(\delta^u_{LL})_{32(31)} = 0$ (a), $\pi/2$ (b), π (c) respectively.

constraints on $(\delta^u_{LL})_{32}$ and $(\delta^u_{LL})_{31}$ and a much weaker constraints (essentially no constrain) on $(\delta^u_{RL})_{32}$ and $(\delta^u_{RL})_{31}$. The resulting bounds on $(\delta^u_{LL})_{32}$ and $(\delta^u_{LL})_{31}$ as functions of the gaugino mass M_2 and the average squark mass $\tilde{m}$ are presented in Tables 5, for $\tan\beta = 10$ and $\mu = 400$ GeV.

The results in Table 5 correspond to positive sign of μ. If one assumed negative sign of μ, the constraints on $|(\delta^u_{LL})_{32}|$ and $|(\delta^u_{LL})_{31}|$ with $\arg(\delta^u_{LR})_{32(31)}$ will be exchanged with the corresponding ones with $\arg(\delta^u_{LL})_{32(31)} + \pi$. Thus, in Table 5, the results of case (a) will be replaced with the results of (c) and vice versa. For larger values of $\tan\beta$, the above constraints will be reduced by the factor $(\tan\beta/10)$. Note also that, because of the $SU(2)$ gauge invariance the soft scalar mass M^2_Q is common for the up and down sectors. Therefore, one gets the following relations between the up and down mass insertions

$$(\delta^d_{LL})_{ij} = \left[V^+_{CKM} \, (\delta^u_{LL}) \, V_{CKM}\right]_{ij} \ . \tag{4.6}$$

Hence,

$$(\delta^d_{LL})_{32} = (\delta^u_{LL})_{32} + \mathcal{O}(\lambda^2) \ . \tag{4.7}$$

As a result, the constraints obtained from the chargino contribution to $b \to s\gamma$ transition on $|(\delta^u_{LL})_{32}|$ can be conveyed to a constraint on $|(\delta^d_{LL})_{32}|$ which equals to $|(\delta^d_{LL})_{23}|$, due to the hermiticity of $(M^2_D)_{LL}$. This is the strongest constraint one may obtain on $|(\delta^d_{LL})_{23}|$, and therefore it should be taken into account in analyzing the LL part of the gluino contribution to the $b \to s$.
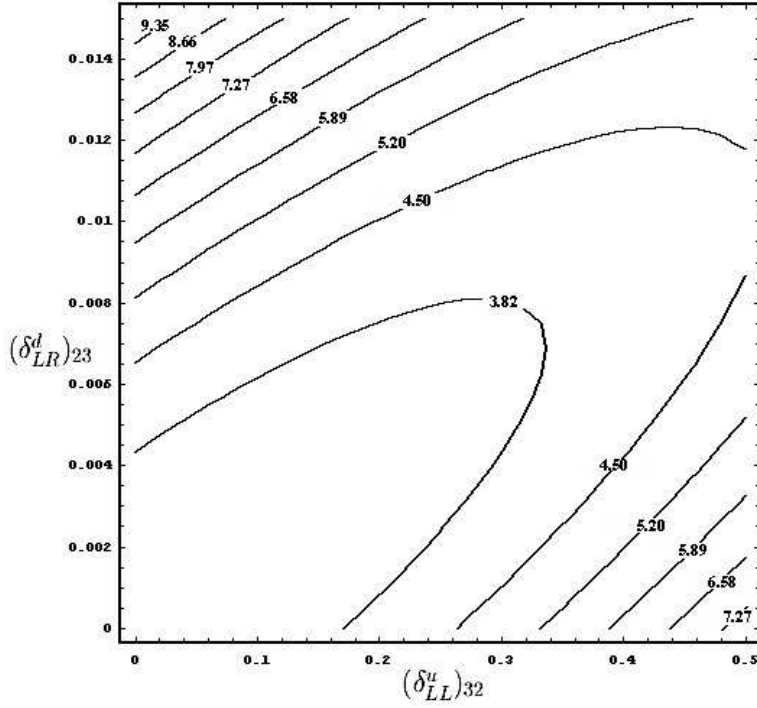
Figure 1: Contour plot for $BR(b \to s\gamma) \times 10^4$ as function of $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$.

Finally we consider the scenario in which both gluino and chargino exchanges are assumed to contribute to $b \to s\gamma$ simultaneously with relevant mass insertions, namely $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$. It is known that these two contributions could give rise to a substantial destructive or constructive interference with the SM amplitude, depending on the relative sign of these amplitudes. Recall that in minimal super-symmetric standard model with the universality assumptions, the gluino amplitude is negligible, since $(\delta^d_{LR})_{23} \lesssim \mathcal{O}(10^{-6})$, and the chargino contribution at large $\tan\beta$ is the only relevant SUSY contribution. In this class of model, depending on the sign of μ the chargino contribution gives destructive interference with the SM result.

In generic SUSY model, the situation is different and the experimental results of the branching ratio of $b \to s\gamma$ can be easily accommodated by any one of these con-tributions. Also since the gluino and the chargino contributions are given in terms of the parameters of the up and down squark sectors, they are, in principle, inde-pendent and could have destructive interference between themselves or with the SM contribution. We stress that we are not interested in any fine tuning region of the parameter space that may lead to a large cancelation. We are rather considering the general scenario with large down and up mass insertions favored by the CP asymme-tries of different B processes. In this case, both gluino and chargino contributions to $b \to s\gamma$ are large and cancelation of order $20 - 50\%$ can take place.

Now, it is clear that the previous constrained obtained on $(\delta_{LR}^d)_{23}$ and $(\delta_{LL}^u)_{32}$ in Tables 4 and 5 will be relaxed. We plot the corresponding results for the correlations between $(\delta_{LR}^d)_{23}$ and $(\delta_{LL}^u)_{32}$ in Fig. 1. Here we consider the relation $(\delta_{LL}^d)_{23} = (\delta_{LL}^u)_{32}$ into account and also set $(\delta_{RL}^u)_{32}$ to zero. The phases of $(\delta_{LR}^d)_{23}$ and $(\delta_{LL}^u)_{32}$ are assumed to be of order $\pi/2$ as favored by the CP asymmetry of $B \to \phi K$. From this plot, we can see that constraints on these mass insertions, particularly $(\delta_{LL}^u)_{32}$ are relaxed.

5. SUSY solution to the $R_c - R_n$ Puzzle

Now we analyze the supersymmetric contributions to the $B \to K\pi$ branching ratio. We will show that the simultaneous contributions from penguin diagrams with chargino and gluino in the loop could lead to a possible solution to the $R_c - R_n$ puzzle. As mentioned in section 3, these penguin contributions have three possible sources of large SUSY contribution to $B \to K\pi$ processes:

1. Gluino mass enhanced $O_{7\gamma}$ and O_{8g} which depend on $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^d)_{23}$.

2. Chargino mass enhanced $O_{7\gamma}$ and O_{8g} which depend on $\tan\beta(\delta_{LL}^u)_{23}$.

3. Right handed stop mass enhanced Z penguin which is given in terms of $(\delta_{RL}^u)_{32}$.

For the same inputs of SUSY parameters that we used above: $m_{\tilde{g}} = 500$ GeV, $m_{\tilde{q}} = 500$ GeV, $m_{\tilde{t}_R} = 150$ GeV, $M_2 = 200$ GeV, $\mu = 400$ GeV, and $\tan\beta = 10$, one finds the following SUSY contributions to the amplitudes of $B \to K\pi$

$$A_{\bar{B}^0 \to \pi^0 \bar{K}^0} \times 10^7 \simeq -9.82 \, i \, \left[(\delta_{LR}^d)_{23} + (\delta_{RL}^d)_{23}\right] + 0.036 \, i \, (\delta_{LL}^u)_{32} - 0.02 \, i \, (\delta_{RL}^u)_{32},$$

$$A_{\bar{B}^0 \to \pi^+ K^-} \times 10^7 \simeq 14.04 \, i \, \left[(\delta_{LR}^d)_{23} + (\delta_{RL}^d)_{23}\right] + 0.06 \, i \, (\delta_{LL}^u)_{32} - 0.001 \, i \, (\delta_{RL}^u)_{32},$$

$$A_{B^- \to \pi^0 K^-} \times 10^7 \simeq 9.9 \, i \, \left[(\delta_{LR}^d)_{23} + (\delta_{RL}^d)_{23}\right] - 0.04 \, i \, (\delta_{LL}^u)_{32} + 0.024 \, i \, (\delta_{RL}^u)_{32},$$

$$A_{B^- \to \pi^- K^0} \times 10^7 \simeq 13.89 \, i \, \left[(\delta_{LR}^d)_{23} + (\delta_{RL}^d)_{23}\right] + 0.05 \, i \, (\delta_{LL}^u)_{32} - 0.006 \, i \, (\delta_{RL}^u)_{32}.$$

It is remarkable that for the amplitudes $A_{\bar{B}^0 \to \pi^0 \bar{K}^0}$ and $A_{B^- \to \pi^0 K^-}$, which suffer from a large discrepancy between their SM values and their experimental measurements, the SUSY contributions have the following features: (i) the effect of $(\delta_{RL}^u)_{32}$ is not negligible as in the other amplitudes, (ii) there can be a distractive interference between the $(\delta_{LR}^d)_{23}$ and $(\delta_{LL}^u)_{32}$ contributions. As we will see below, these two points are important in saturating the experimental results by supersymmetry. Also note that the effect of gluino contribution through $O_{7\gamma}$ is very small and the contribution of $(\delta_{LR}^d)_{23}$ is mainly due to O_{8g}. However the chargino effect of $O_{7\gamma}$ can be enhanced by $\tan\beta$.
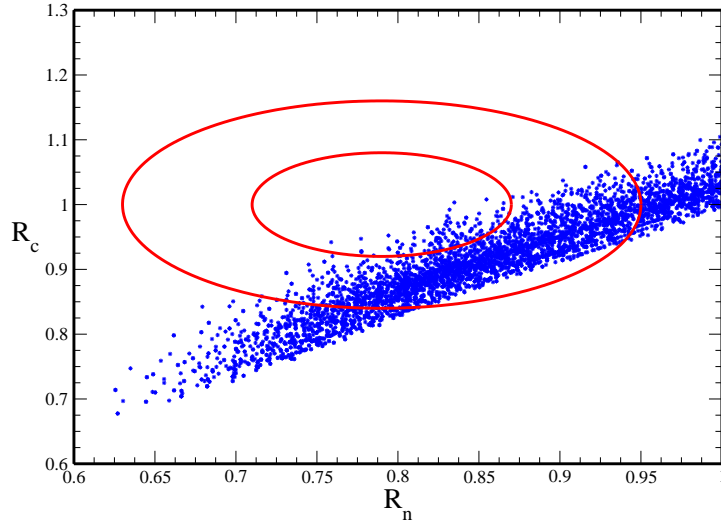
Figure 2: $R_c - R_n$ correlation in SUSY models with $|(\delta^u_{LR})_{32}| \simeq 1$, $|(\delta^d_{LR})_{23}| \in [0.001, 0.01]$ and $|(\delta^u_{LL})_{32}| \in [0.1, 1]$; see the text for the other parameters. The small and large ellipses correspond to 1σ and 2σ experimental results, respectively.

We present our numerical results for the correlation between the total contributions (SM+SUSY) to the R_n and R_c in Fig. 2. We have scanned over the relevant mass insertions: $(\delta^u_{LL})_{32}$, $(\delta^d_{LR})_{23}$ and $(\delta^u_{RL})_{32}$, since we have assumed $(\delta^u_{LL})_{32} \simeq (\delta^d_{LL})_{23}$ and $(\delta^d_{LR})_{23} \simeq (\delta^d_{RL})_{23}$. We considered $|(\delta^u_{LL})_{32}| \in [0.1, 1]$, $|(\delta^d_{LR})_{23}| \in [0.001, 0.01]$, $\arg[(\delta^u_{LL})_{32}] \in [-\pi, \pi]$, $\arg[(\delta^d_{LR})_{23}] \simeq \pi/3$ (which is preferred by $S_{\phi K_S}$), and $(\delta^u_{RL})_{32} = 1$ (in order to maximize the difference between R_n and R_c). As can be seen from the results in Fig. 2, the experimental results of R_n and R_c at 2σ can be naturally accommodated by the SUSY contributions. However, the results at 1σ can be only obtained by a smaller region of parameter space. In fact, the values of R_c is predicted to be less than one for the most of the parameter space. Therefore, it will be nice accordance with SUSY results if the experimental result of R_c goes down.

In order to understand the results in Fig. 2 and the impact of the SUSY on the correlation between R_n and R_c, we extend the parametrization introduced in section 2 for the relevant amplitudes by including the SUSY contribution [8]. In this case, Eqs.(2.24) can be written as

$$A_{B^- \to \pi^- \overline{K}^0} = \lambda_c A_{\pi \bar{K}} P \left[e^{i\theta_P} + r_A e^{i\delta_A} e^{-i\gamma} \right] \tag{5.1}$$

$$\sqrt{2} A_{B^- \to \pi^0 K^-} = \lambda_c A_{\pi \bar{K}} P [e^{i\theta_P} + \left(r_A e^{i\delta_A} + r_C e^{i\delta_C} \right) e^{-i\gamma} + r_{EW} e^{i\theta_{EW}} e^{i\delta_{EW}}] \tag{5.2}$$

$$A_{\overline{B}^0 \to \pi^+ K^-} = \lambda_c A_{\pi \overline{K}} P \left[e^{i\theta_P} + \left(r_A e^{i\delta_A} + r_T e^{i\delta_T} \right) e^{-i\gamma} + r^C_{EW} e^{i\theta_{EW}} e^{i\delta^C_{EW}} \right], \tag{5.3}$$

$$-\sqrt{2} A_{\overline{B}^0 \to \pi^0 \overline{K}^0} = \lambda_c A_{\pi \overline{K}} P \Big[e^{i\theta_P} + \left(r_A e^{i\delta_A} + r_T e^{i\delta_T} - r_C e^{i\theta_C} e^{i\delta_C} \right) e^{-i\gamma} + r^C_{EW} e^{i\theta^C_{EW}} e^{i\delta^C_{EW}}$$
$$- r_{EW} e^{i\theta_{EW}} e^{i\delta_{EW}} \Big]. \tag{5.4}$$

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The parameters $\delta_A, \delta_C, \delta_T, \delta_{EW}, \delta_{EW}^C$ and $\theta_P, \theta_{EW}, \theta_{EW}^C$ are the CP conserving (strong) and the CP violating phase, respectively. Note that the parameters P, r_{EW}, r_{EW}^C are now defined as

$$Pe^{i\theta_P}e^{i\delta_P} = \alpha_4^c - \frac{1}{2}\alpha_{4,EW}^c + \beta_3^c + \beta_{3,EW}^c,$$

$$r_{EW}e^{i\theta_{EW}}e^{i\delta_{EW}} = \left[\frac{3}{2}(R_{K\pi}\alpha_{3,EW}^c + \alpha_{4,EW}^c)\right]/P,$$

$$r_{EW}^C e^{i\theta_{EW}^C}e^{i\delta_{EW}^C} = \left[\frac{3}{2}(\alpha_{4,EW}^c - \beta_{3,EW}^c)\right]/P. \tag{5.5}$$

First let us include some assumptions to simplify our formulae. As is mentioned before, $\alpha_4^p, \alpha_{3,EW}^p, \alpha_{4,EW}^p, \beta_3^p, \beta_{3,EW}^p, \beta_{4,EW}^p$ receive SUSY contributions through the Wilson coefficients. The upper index p takes both u and c, however the contribution with u index is always suppressed by the factor $\epsilon_{KM} \simeq 0.018$ so that its SUSY contributions can be safely neglected comparing to the one with the index c. As a result, $(r_A e^{i\delta_A}), (r_C e^{i\delta_C})$ and $(r_T e^{i\delta_T})$ receive a correction of a factor $1/|1 + \frac{P^{\mathrm{SUSY}}}{P^{\mathrm{SM}}}|$.

Secondly we assume that the strong phase for SM and SUSY are the same. We found that this is a reasonable assumption in QCD approximation in which the main source of the strong phase comes from hard spectator and weak annihilation diagrams. This leads us to the following parametrization:

$$Pe^{i\delta_P}e^{i\theta_P} = P^{\mathrm{SM}}e^{i\delta_P}(1 + ke^{i\theta'_P}) \tag{5.6}$$

$$r_{EW}e^{i\delta_{EW}}e^{i\theta_{EW}} = (r_{EW})^{\mathrm{SM}}e^{i\delta_{EW}}(1 + le^{i\theta'_{EW}}) \tag{5.7}$$

$$r_{EW}^C e^{i\delta_{EW}^C}e^{i\theta_{EW}^C} = (r_{EW}^C)^{\mathrm{SM}}e^{i\delta_{EW}^C}(1 + me^{i\theta_{EW}^{C'}}). \tag{5.8}$$

where

$$ke^{i\theta'_P} \equiv \frac{(\alpha_4^c - \frac{1}{2}\alpha_{4,EW}^c + \beta_3^c + \beta_{3,EW}^c)_{\mathrm{SUSY}}}{(\alpha_4^c - \frac{1}{2}\alpha_{4,EW}^c + \beta_3^c + \beta_{3,EW}^c)_{\mathrm{SM}}}, \tag{5.9}$$

$$le^{i\theta'_q} \equiv \frac{(R_{K\pi}\alpha_{3,EW}^c + \alpha_{4,EW}^c)_{\mathrm{SUSY}}}{(R_{K\pi}\alpha_{3,EW}^c + \alpha_{4,EW}^c)_{\mathrm{SM}}}, \tag{5.10}$$

$$me^{i\theta'_{qC}} \equiv \frac{(\alpha_{4,EW}^c - \beta_{3,EW}^c)_{\mathrm{SUSY}}}{(\alpha_{4,EW}^c - \beta_{3,EW}^c)_{\mathrm{SM}}} \tag{5.11}$$

The index SM (SUSY) mean to keep only SM (SUSY) Wilson coefficients in $\alpha_{i(,EW)}^p$ and $\beta_{i(,EW)}^p$. Using these parameters, we also have

$$r_A e^{i\delta_A} = \frac{(r_A e^{i\delta_A})_{\mathrm{SM}}}{|1 + ke^{i\theta'_P}|}, \quad r_C e^{i\delta_C} = \frac{(r_C e^{i\delta_C})_{\mathrm{SM}}}{|1 + ke^{i\theta'_P}|}, \quad r_T e^{i\delta_T} = \frac{(r_T e^{i\delta_T})_{\mathrm{SM}}}{|1 + ke^{i\theta'_P}|} \tag{5.12}$$

Now let us investigate the $R_c - R_n$ puzzle. We shall follow the standard procedure; to simplify and expand the formulae. Considering the numbers obtained above, we shall simplify our formulae by assuming

1. the strong phases are negligible, i.e., $\delta_P, \delta_A, \delta_C, \delta_{EW}, \delta_{EW}^C$ are all zero.

2. the annihilation tree contribution is negligible, i.e. $r_A \simeq 0$

3. the color suppressed tree contribution is negligible, i.e. $r_C e^{i\delta_C} \sim r_T e^{i\delta_T}$.

Using these assumptions, we expand R_c, R_n and $R_c - R_n$. We expand in terms of r_T and r_{EW} and r_{EW}^C up to the second order. As a result, we obtain

$$R_c \simeq 1 + r_T^2 - 2r_T \cos(\gamma + \theta_P) + 2r_{EW} \cos(\theta_P - \theta_{EW}) - 2r_T r_{EW} \cos(\gamma + \theta_{EW}) \quad (5.13)$$

$$R_c - R_n \simeq 2r_T r_{EW} \cos(\gamma + 2\theta_P - \theta_{EW}) - 2r_T r_{EW}^C \cos(\gamma + 2\theta_{EW} - \theta_{EW}^C) \quad (5.14)$$

Now, let us find the configuration which lead to $R_c - R_n > 0.2$. Looking at Eq. (5.14), we can find that in general, the larger the values of r_T, r_{EW} and r_{EW}^C are, the larger the splitting between R_c and R_n we would acquire. The phase combinations $\theta_P - \theta_{EW}$ and $\theta_P + \gamma$ also play an important role. The possible solution of $R_c - R_n$ puzzle by enhancing r_{EW}, which we parameterize as l, has been intensively studied in the literature [9]. As we will see in the following, r_T can also be enhanced due to the factor $ke^{i\theta'_P}$ which contributes destructively against the SM and diminish P. However, since P is the dominant contribution to the $B \to K\pi$ process, the branching ratio is very sensitive to $ke^{i\theta'_P}$. Therefore, we are allowed to vary $ke^{i\theta'_P}$ only in a range of the theoretical uncertainty of QCD factorization, which gives about right sizes of the $B \to K\pi$ branching ratios. As showed in Ref.[8], we would be able to reduce P at most by 30 %, which can be easily compensated by the error in the transition form factor $F^{B\to\pi,K}$.

Considering the tiny effect from the second term in Eq. (5.14), in order to achieve $R_c - R_n \gtrsim 0.2$, we need $r_T r_{EW}$ larger than about 0.1 or equivalently, r_{EW} larger than about 0.5 with r_T^{SM}. In Ref.[8], it was emphasized that with $k = 0$, one needs $l \gtrsim 2$ to reproduce the experimental values while an inclusion of a small amount of k lowers this bound significantly. For the SUSY parameters that we have considered above, the following results for our SUSY parameters k, l, and m are obtained

$$ke^{i\theta_P} = -0.0019 \tan\beta (\delta_{LL}^u)_{32} - 35.0(\delta_{LR}^d)_{23} + 0.061(\delta_{LR}^u)_{32} \quad (5.15)$$

$$le^{i\theta_q} = 0.0528 \tan\beta (\delta_{LL}^u)_{32} - 2.78(\delta_{LR}^d)_{23} + 1.11(\delta_{LR}^u)_{32} \quad (5.16)$$

$$me^{i\theta_{qC}} = 0.134 \tan\beta (\delta_{LL}^u)_{32} + 26.4(\delta_{LR}^d)_{23} + 1.62(\delta_{LR}^u)_{32} \quad (5.17)$$

Note that we do not consider $(\delta_{23}^d)_{RL}$ here but it is the same as $(\delta_{LR}^d)_{23}$ with an opposite sign (see also [7]). Let us first discuss the contributions from a single mass insertion $(\delta_{LL}^u)_{32}, (\delta_{LR}^d)_{23}$ or $(\delta_{LR}^u)_{32}$ to $\{k, l, m\}$; keeping only one mass insertion and switching off the other two. In this case, one finds that the maximum value of

$\{k, l, m\}$ with $|(\delta^u_{LR})_{32}| = 1$ is $\{k, l, m\} = \{0.061, 1.11, 1.62\}$. Thus, in this case where k is almost negligible, we would need $l \simeq 2$ to explain the experimental data. We have a chance to enlarge the coefficients for $(\delta^u_{LR})_{32}$ by, for instance, increasing the averaged squark mass $\tilde{m}_{\tilde{q}}$. However, even if we choose $\tilde{m}_{\tilde{q}} = 5$ TeV, we find that l is increased only by 20 to 30 %. The maximum contributions from $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ are found to be $\{k, l, m\} = \{0.18, 0.014, 0.13\}$ and $\{0.0019, 0.053, 0.13\}$, which are far too small to explain the experimental data. The coefficients for $(\delta^d_{LR})_{23}$ depend on the overall factor $1/\tilde{m}_{\tilde{q}}$ and on also the variable of the loop function $x = m_{\tilde{g}}/\tilde{m}_{\tilde{q}}$ and we found that $m_{\tilde{g}} = \tilde{m}_{\tilde{q}} = 250$ GeV can lead to 100 % increase. However, the value of l is still too small to deviate $R_c - R_n$ significantly. As a whole, it is extremely difficult to have $R_c - R_n \gtrsim 0.2$ from a single mass insertion contribution.

Let us try to combine two main contributions, $(\delta^d_{LR})_{23}$ and $(\delta^u_{LR})_{32}$ terms. Using the previous input parameters and including the $b \to s\gamma$ constraint $|(\delta^d_{LR})_{23}|$, the maximum value is found to be $\{k, l, m\} = \{0.24, 1.12, 1.48\}$. In this case, it is easy to check that the experimental data are not reproduced very well [8]. As discussed above, for a large value of the averaged squark masses, l increases while k decreases. On the contrary, k also depends on the ratio of gluino and squark masses. Hence we need to optimize these masses so as to increase k and l simultaneously. For instance, with $m_{\tilde{g}} = 250$ GeV and $\tilde{m}_{\tilde{q}} = 1$ TeV, we obtain $\{k, l, m\} = \{0.30, 1.36, 1.90\}$ which leads to a result within the experimental bounds of R_c and R_n. Finally, we consider the case with the three non-zero mass insertions. The main feature of this scenario is that we expect a relaxation of the constraints on $|\tan\beta \times (\delta^u_{LL})_{32}|$ and $|(\delta^d_{LR})_{23}|$ from the cancelation between $(\delta^d_{LR})_{23}$ and $(\delta^u_{LL})_{32}$ contributions to $b \to s\gamma$. Under this circumstance, we observe much larger $R_c - R_n$ for various combination of the phases in this scenario.

6. SUSY contributions to the CP asymmetry of $B \to K\pi$

We start this section by summarizing our convention for CP asymmetry in $B \to K\pi$ processes. The time dependent CP asymmetry for $B \to K\pi$ can be described by

$$A_{K\pi}(t) = A_{K\pi}\cos(\Delta M_{B_d}t) + S_{K\pi}\sin(\Delta M_{B_d})t, \qquad (6.1)$$

where $A_{K\pi}$ and $S_{K\pi}$ represent the direct and the mixing CP asymmetry respectively and they are given by

$$A_{K\pi} = \frac{|\bar{\rho}(K\pi)|^2 - 1}{|\bar{\rho}(K\pi)|^2 + 1}, \qquad S_{K\pi} = \frac{2Im(\bar{\rho}(K\pi))}{|\bar{\rho}(K\pi)|^2 + 1}, \qquad (6.2)$$

where $\bar{\rho}(K\pi) = e^{-i\phi_B}\frac{\bar{A}(K\pi)}{A(K\pi)}$. The phase ϕ_B is the phase of M_{12}, the $B^0 - \bar{B}^0$ mixing amplitude. The $A(K\pi)$ and $\bar{A}(K\pi)$ are the decay amplitudes for B^0 and $\bar{B}^0$ to $K\pi$, respectively.

The SM predicts that the direct and mixing asymmetry of $B \to K\pi$ decay are given by

$$S_{K\pi} = \sin 2\beta, \qquad C_{K\pi} = 0. \tag{6.3}$$

The recent measurements of the CP asymmetries in $B \to K\pi$, reported in Table 1, show significant discrepancies with the SM predictions. As mentioned above, SUSY can affect the results of the CP asymmetries in B decay, due to the new source of CP violating phases in the corresponding amplitude. Therefore, deviation on CP asymmetries from the SM expectations can be sizeable, depending on the relative magnitude of the SM and SUSY amplitudes. In this respect, SUSY models with non-minimal flavor structure and new CP violating phases in the squark mass matrices, can generate large deviations in the $B \to K\pi$ asymmetry. In this section we present and discuss our results for SUSY contributions to the direct and mixing CP asymmetries in $B \to K\pi$.

6.1 SUSY contributions to the direct CP asymmetry in $B \to K\pi$

Using the general parametrization of the decay amplitudes of $B \to K\pi$ given in Eqs.(5.1-5.4), one can write the direct CP asymmetries $A_{K\pi}^{CP}$ as follows:

$$
\begin{aligned}
A_{K^-\pi^+}^{CP} \simeq\ & 2r_T \sin\delta_T \sin(\theta_P + \gamma) + 2r_{EW}^C \sin\delta_{EW}^C \sin(\theta_P - \theta_{EW}^c) - r_T^2 \sin 2\delta_T \sin 2(\theta_P + \gamma) \\
& + 2r_T r_{EW}^C \sin(\delta_{EW}^C - \delta_T)\sin(\theta_{EW}^C + \gamma) - 4r_T r_{EW}^C \sin\delta_{EW}^C \sin(\theta_P - \theta_{EW}^C)\cos\delta_T \\
& \quad \cos(\theta_P + \gamma) - 4r_T r_{EW}^C \sin\delta_T \sin(\theta_P + \gamma)\cos\delta_{EW}^c \cos(\theta_P - \theta_{EW}^C), \tag{6.4}
\end{aligned}
$$

$$A_{\bar{K}^0\pi^-}^{CP} \simeq 2r_A \sin\delta_A \sin(\theta_P + \gamma), \tag{6.5}$$

$$A_{\bar{K}^0\pi^0}^{CP} \simeq 2r_{EW}^C \sin\delta_{EW}^C \sin(\theta_P - \theta_{EW}^C) - 2r_{EW} \sin\delta_{EW} \sin(\theta_P - \theta_{EW}), \tag{6.6}$$

$$
\begin{aligned}
A_{K^-\pi^0}^{CP} \simeq\ & 2r_T \sin\delta_T \sin(\theta_P + \gamma) - 2r_{EW} \sin\delta_{EW} \sin(\theta_P - \theta_{EW}) - r_T^2 \sin 2\delta_T \sin 2(\theta_P + \gamma) \\
& - 2r_T r_{EW} \sin(\delta_{EW} - \delta_T)\sin(\theta_{EW} + \gamma) - 4r_T r_{EW} \sin\delta_{EW} \sin(\theta_P - \theta_{EW})\cos\delta_T \\
& \quad \cos(\theta_P + \gamma) - 4r_T r_{EW} \sin\delta_T \sin(\theta_P + \gamma)\cos\delta_{EW} \cos(\theta_P - \theta_{EW}). \tag{6.7}
\end{aligned}
$$

From these expressions, it is clear that if we ignore the strong phases, then the direct CP asymmetries would vanish. However, Belle and BaBar collaborations observed non-zero values for the $A_{K\pi}^{CP}$, thus we should consider non-vanishing strong phases in this analysis. It is also remarkable that the leading contributions to the direct CP asymmetries are given by the linear terms of $r_i \equiv r_T, r_A, r_{EW}, r_{EW}^C$, unlike the difference $R_c - R_n$ which receives corrections of order $r_i r_j$. As in the previous section, we have assumed that the color suppressed contributions are negligible $i.e.$, $r_C e^{i\delta_C} =$

$r_T e^{i\delta_T}$ and we have neglected terms of order r_i^2 except for r_T which is typically larger than r_{EW}, r_{EW}^C, and r_A.

The rescattering effects parameterized by r_A are quite small $\left(r_A^{SM} \simeq \mathcal{O}(0.01)\right)$ therefore the CP asymmetry in the decays $B^\pm \to K^0 \pi^\pm$ is expected to be very small as can be easily seen from Eq.(6.5). This result is consistent with the experimental measurements reported in Table 1. The sign of this asymmetry will depend on the relative sign of $\sin \delta_A$ and $\sin(\theta_P + \gamma)$. Note that the value of the angle γ is fixed by the CP asymmetry in $B \to \pi\pi$ to be of order $\pi/3$. The angle θ_P can also be determined from the CP asymmetry $S_{\phi(\eta')K}$.

In the SM, the parameters r_A, r_{EW}^C are much smaller than r_T, r_{EW} and $\theta_P = 0$, therefore the following relation among the direct CP asymmetries $A_{K\pi}^{CP}$ is obtained

$$A_{K^-\pi^0}^{CP} \gtrsim A_{K^-\pi^+}^{CP} \gtrsim A_{K^0\pi^0}^{CP} > A_{K^0\pi^-}^{CP}.$$

This relation is in agreement with the numerical results listed in Table 3 for the direct CP asymmetries in the SM with $\rho_{A,H}, \phi_{A,H} \simeq 1$. To change this relation among the CP asymmetries and to get consistent correlations with experimental measurements, one should enhance the electroweak penguin contributions to $\bar{B}^0 \to K^-\pi^+$ decay amplitude, parameterized by r_{EW}^C. Furthermore, a non-vanishing value of θ_P, which is also required to account for the recent measurements of $S_{\phi K_S}$ and $S_{\eta' K_S}$, is favored in order to obtain $A_{K^-\pi^+}^{CP} > A_{K^-\pi^0}^{CP}$. It is worth mentioning that in the SM and due to the fact that $\theta_P = 0$ the second term in Eq.(6.4) and Eq.(6.7) give destructive and constructive interferences respectively with first terms. Thus one finds $A_{K^-\pi^0}^{CP}$ is larger than $A_{K^-\pi^+}^{CP}$. In SUSY models, the gluino contribution leads to a large value of θ_P and depending on the sign of this angle the parameter r_T could be enhanced or reduced, see Eq.(5.12). As will be seen below, in this case we can explain the CP asymmetry results with moderate values of the electroweak penguin parameter r_{EW}^C. Note that in other models studied in the literatures, the value of this parameter is required to be larger than one in order to account for the CP asymmetry results.

Now let us discuss the SUSY contribution to the CP asymmetries $A_{K\pi}^{CP}$. As can be seen from Table 1 that the experimental measurements of $A_{K^0\pi^0}^{CP}$ suffer from a large uncertainty. It turns out that it is very easy to have the SUSY results for this asymmetry within the range of 2σ measurements. Thus, this decay mode is not useful in constraining the SUSY parameter space and can be ignored in our discussion for the correlation among the CP asymmetries of $B \to K\pi$ in generic SUSY models.

We will consider, as in the previous section, three scenarios with a single mass insertion, two mass insertions, and three mass insertions. In the first case, if we consider the contribution due to the mass insertion $(\delta_{LR}^u)_{32}$ the maximum values of

$\{k, l, m\}$ are given by $\{0.061, 1.11, 1.62\}$. While from $(\delta_{LR}^d)_{23}$ and $(\delta_{LL}^u)_{32}$ one finds that the maximum values of $\{k, l, m\}$ are $\{0.18, 0.014, 0.13\}$ and $\{0.0019, 0.053, 0.13\}$ respectively. Note that k is almost negligible in the case of dominant chargino contribution which depends on $(\delta_{LL}^u)_{32}$ and $(\delta_{LR}^u)_{32}$ and can be significantly enhanced by the gluino contribution that depends on $(\delta_{LR}^d)_{23}$ as emphasized in Ref.[5]. Also from Eqs.(5.7,5.8), and (5.12), one finds

$$r_{EW} = r_{EW}^{SM}(1 + l^2 + 2l\cos\theta'_{EW})^{1/2}, \tag{6.8}$$

$$r_{EW}^c = (r_{EW}^C)^{SM}(1 + m^2 + 2m\cos\theta_{EW}^{C'})^{1/2}, \tag{6.9}$$

$$r_T = \frac{r_T}{|1 + ke^{i\theta'_P}|}. \tag{6.10}$$

Since $(r_{EW}^C)^{SM} \simeq 0.01$, the enhancement of r_{EW}^C remains quite limited in SUSY models and it is impossible to enhance it to be of order one. Hence, the contribution of r_{EW}^C to $A_{K^-\pi^+}^{CP}$ is negligible respect to the contribution of r_{EW} to $A_{K^-\pi^0}^{CP}$. To overcome this problem and get the desired relation between $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$ a kind of cancelation between r_T and r_{EW} contributions to $A_{K^-\pi^0}^{CP}$ is required. Such cancelation can be obtained naturally without fine tuning the parameters if $r_T \sim r_{EW}$, i.e. the total value of $r_T < r_T^{SM}$. This could happen if k is not very small. Therefore, one would expect that the scenarios with dominant chargino contribution, where $k = 0.061$ or $k = 0.0019$ will not be able to saturate the experimental results of $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$ simultaneously. This observation is confirmed in Fig. 3(top-left), where the results of $A_{K^-\pi^+}^{CP}$ is potted versus the results of $A_{K^-\pi^0}^{CP}$ for $\{k, l, m\} = \{0.061, 1.11, 2.62\}$ and the other parameters vary as follows: $\delta_i \equiv -\pi, \pi/2$, and π. The angles θ_{EW} and $\theta_{EW}^C \in [-\pi, \pi]$. Also θ_P is assumed to be in the region $[\pi/4, \pi/2]$. Note that in this plot we have taken the $A_{K^0\pi^-}^{CP}$ as constraint. Thus all the points in the plot correspond to consistent values of $A_{K^0\pi^-}^{CP}$ with the experimental results.

Now we consider the second scenario with dominant gluino contribution, i.e., $(\delta_{LR}^d)_{23} \simeq 0.005e^{i\pi/3}, (\delta_{LL}^u)_{32} = (\delta_{RL}^u)_{32} = 0$. In this case, one finds that the maximum values of $\{k, l, m\}$ are give by $\{k, l, m\} = \{0.18, 0.014, 0.13\}$, hence r_T is reduced from $r_T^{SM} \simeq 0.2$ to $r_T \simeq 0.12$, while r_{EW} and r_{EW}^C approximately remain the same as in the SM. In Fig. 3(top-right) we plot the CP asymmetries $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$ in this scenario, with varying the relevant parameter as before. It is remarkable that large number of points of the parameter space can simultaneously accommodate the experimental results of these CP asymmetries. It is slightly surprising to get the values of the CP asymmetries $A_{K\pi}^{CP}$ within the experimental range, i.e., $A_{K^-\pi^+}^{CP} \in [-0.075, -0.151]$ and $A_{K^-\pi^0}^{CP} \in [-0.04, 0.12]$, by just one mass insertion in dominant gluino models. This is contrary to the $R_c - R_n$ results which need gluino and chargino combination in order to be within the experimental range. This result
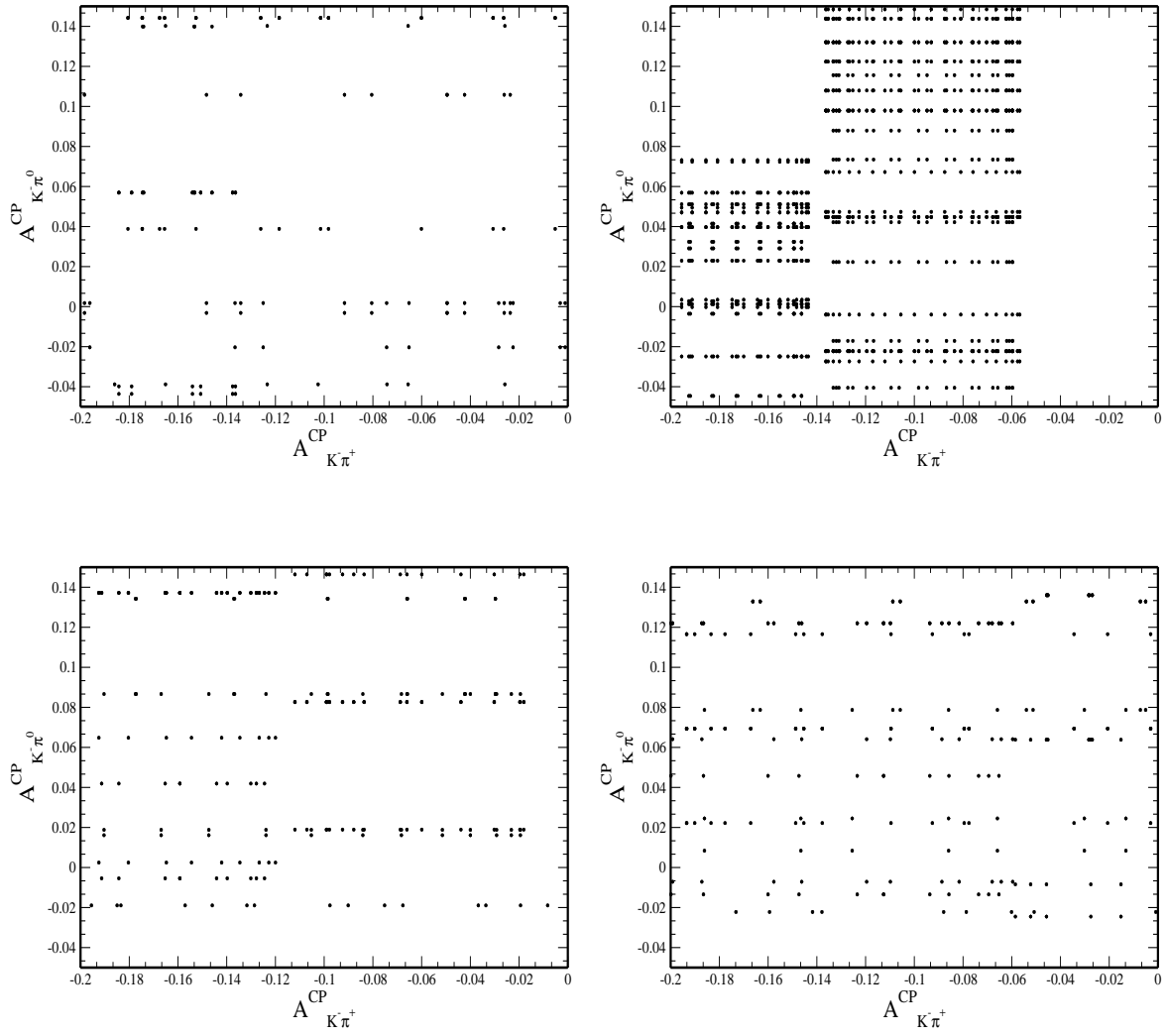
Figure 3: CP asymmetry of $B \rightarrow K^- \pi^+$ versus CP asymmetry of $B \rightarrow K^- \pi^0$ for $\{k, l, m\} = \{0.061, 1.11, 2.62\}, \{0.18, 0.014, 0.13\}, \{0.24, 1.12, 1.48\}, \{0.32, 0.95, 2.26\}$ respectively from left to the right and top to bottom. Strong phases $\delta_i \equiv -\pi, \pi/2, \pi$ and CP violating phases θ_{EW} and θ_{EW}^C reside between $-\pi$ and π. Finally θ_P is assumed to be in the region $[\pi/4, \pi/2]$.

can be explained by the cancelation that occurs in $A_{K^- \pi^0}^{CP}$ between the r_T and r_{EW} contributions and the negligible effect of r_{EW}^C to $A_{K^- \pi^+}^{CP}$.

To be more quantitative, let us consider the following example where $(\delta_{LR}^d)_{23} \simeq 0.005 \, e^{i\pi/3}$ and $(\delta_{LL}^u)_{32} = (\delta_{RL}^u)_{32} = 0$. In this case one get $r_T = 0.12$, $r_{EW} = 0.13$ and $r_{EW}^C = 0.01$. Therefore, the main contribution to $A_{K^- \pi^+}^{CP}$ is due to the linear term in r_T which is $r_T \sin \delta_T \sin(\theta_P + \gamma)$. With $\theta_P \sim \pi/3$ and $\delta_T \sim -\pi/4$, this contribution leads to $A_{K^- \pi^+}^{CP} \simeq -0.113$. Since r_T gives the same contributions to

$A_{K^-\pi^0}^{CP}$ a significant positive contribution from r_{EW} is required to change the $A_{K^-\pi^0}^{CP}$ and make it positive. With $r_{EW} = 0.13$, the $A_{K^-\pi^0}^{CP}$ is approximately given by $A_{K^-\pi^0}^{CP} \simeq -0.113 + 0.26 \sin \delta_{EW} \sin(\theta_P - \theta_{EW})$. It is worth mentioning that although θ'_P and θ'_{EW} are equal in case of single mass insertion, the value of θ_P and θ_{EW} are different due to the different value of k and l. In this example, it turns out that $\theta_P - \theta_{EW} \sim \pi/9$. Hence, one gets $A_{K^-\pi^0}^{CP} \simeq -0.113 + 0.22 \sin \delta_{EW}$. So that for $\delta_{EW} \sim \pi/4$, one finds $A_{K^-\pi^0}^{CP} \simeq 0.04$ which is the central value of the experimental measurements reported in Table 1.

We turn to the contributions from two mass insertions: $(\delta_{LR}^d)_{23}$ and $(\delta_{RL}^u)_{32}$, which reflect simultaneous contributions from the penguin diagrams with chargino and gluino in the loop. Applying the $b \to s\gamma$ constraints on these mass insertions, the maximum values of $\{k, l, m\}$ is found to be $\{0.24, 1.12, 1.148\}$. In this case we obtain $r_T = 0.11$, $r_{EW} = 0.54$ and $r_{EW}^C = 0.06$. Therefore, the CP asymmetry $A_{K^-\pi^0}^{CP}$ is dominated by r_{EW} contribution and in order to get $A_{K^-\pi^0}^{CP}$ of order $\mathcal{O}(0.04)$, a small value of the strong phase δ_{EW} should be used. This makes the possibility of saturating the results of $A_{K^-\pi^+}^{CP}$ and $A_{K^-\pi^0}^{CP}$ is less possible than the previous case. In Fig. 3(bottom-left), we present the results of this scenario for the same set of input parameters used before. This figure confirms our expectation and it can be easily seen that it has less points of the parameter space that account for the experimental results of the CP asymmetries than Fig. 3(top-right). Note also that with two mass insertions, the phases θ'_P and θ'_{EW} can be considered independent, hence the angles θ_P and θ_{EW} are also independent.

Finally we consider the case of three non-vanishing mass insertions: $(\delta_{LR}^d)_{23}$, $(\delta_{RL}^u)_{32}$ and $(\delta_{LL}^u)_{32}$. Including the $b \to s\gamma$ constraints we find that maximum value of $\{k, l, m\}$ is found to be $\{0.32, 0.95, 2.26\}$. The corresponding values of r_i are $r_T = 0.10$, $r_{EW} = 0.48$ and $r_{EW}^C = 0.09$. It is clear that r_T and r_{EW} are slightly changed than the previous scenario, while r_{EW}^C is enhanced a bit. In this case, it will be easier to accommodate for $A_{K^-\pi^0}^{CP}$. The numerical results for this scenario are given in Fig. 3(bottom-right) for the same set of parameter space used in previous cases. As can be seen from this figure, the probability of accommodating the experimental results of different CP asymmetries in this class of models is higher than it in models with two mass insertions. However, it remains that the model with dominated gluino contributions provides the largest possibility of saturating the experimental results of CP asymmetries of $B \to K\pi$.

6.2 SUSY contributions to the mixing CP asymmetry in $B \to K^0\pi^0$

We turn our attention, now, to the mixing CP asymmetry of $B \to K^0\pi^0$. As men-

tioned before, this decay is dominated by $b \to s$ penguin. Thus, within the SM, the CP asymmetry $S_{K^0\pi^0}$ should be very close to the value of $\sin 2\beta \simeq 0.73$. However, the current experimental measurements summarized in Table 1 show that $S_{K^0\pi^0}$ is lower than the expected value of $\sin 2\beta$, namely

$$S_{K^0\pi^0} \simeq 0.34 \pm 0.28. \tag{6.11}$$

In this section we aim to interpret this discrepancy in terms of supersymmetry contributions. It is useful to parameterize the SUSY effects by introducing the ratio of SM and SUSY amplitudes as follows

$$\left(\frac{A^{\text{SUSY}}}{A^{\text{SM}}}\right)_{K\pi} \equiv R_\pi \, e^{i\theta_\pi} \, e^{i\delta_\pi}, \tag{6.12}$$

where R_π stands for the absolute value of $\left|\frac{A^{\text{SUSY}}(B \to K^0\pi^0)}{A^{\text{SM}}(B \to K^0\pi^0)}\right|$ and the angle θ_π is the SUSY CP violating phase. The strong (CP conserving) phase δ_π is defined by $\delta_\pi = \delta_\pi^{SM} - \delta_\pi^{SUSY}$. This parametrization is analogously for those of $S_{K\phi}$ and $S_{K\eta'}$ [5, 7]. Using this parametrization, one finds that the mixing CP asymmetry $S_{K^0\pi^0}$ in Eq.(6.2) takes the following form

$$S_{K^0\pi^0} = \frac{\sin 2\beta + 2R_\pi \cos\delta_\pi \sin(\theta_\pi + 2\beta) + R_\pi^2 \sin(2\theta_\pi + 2\beta)}{1 + 2R_\pi \cos\delta_\pi \cos\theta_\pi + R_\pi^2}. \tag{6.13}$$

Assuming that the SUSY contribution to the amplitude is smaller than the SM one *i.e.* $R_\pi \ll 1$, one can simplify the above expressions as:

$$S_{K^0\pi^0} = \sin 2\beta + 2\cos 2\beta \sin\theta_\pi \cos\delta_\pi R_\pi + \mathcal{O}(R_\pi^2). \tag{6.14}$$

In order to reduce $S_{K^0\pi^0}$ smaller than $\sin 2\beta$, the relative sign of $\sin\theta_\pi$ and $\cos\delta_\pi$ has to be negative. If one assumes that $\sin\theta_\pi \cos\delta_\pi \simeq -1$, then $R_\pi \gtrsim 0.2$ is required in order to get $S_{K^0\pi^0}$ within 1σ of the experimental range.

In the QCDF approach, the decay amplitude of $B \to K^0\pi^0$ is given by Eq.(2.8). As in the case of $B \to \phi(\eta')K$ [5], we will provide the numerical parametrization of this amplitude in terms of the Wilson coefficients $\mathbf{C_i}$ and $\tilde{\mathbf{C}}_\mathbf{i}$ defined according to the parametrization of the effective Hamiltonian in Eq.(2.1)

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_i \left\{ \mathbf{C}_i Q_i + \tilde{\mathbf{C}}_i \tilde{Q}_i \right\}, \tag{6.15}$$

where the operators basis Q_i and $\tilde{Q}_i$ are the same ones of Eq.(2.1). By fixing the hadronic parameters with their center values as in Table 1 of Ref.[11], we obtain

$$A(B \to K^0\pi^0) = -i\frac{G_F}{\sqrt{2}} m_B^2 F_+^{B\to K} f_\pi \sum_{i=1..10,7\gamma,8g} H_i(\pi)(\mathbf{C}_i - \tilde{\mathbf{C}}_i), \tag{6.16}$$

where

$$H_1(\pi) \simeq -0.7 + 0.0003i,$$

$$H_2(\pi) \simeq -0.21 + 0.037i - 0.006X_H,$$

$$H_3(\pi) \simeq 0.22 - 0.076i + 0.0045X_A + 0.0003X_A^2 + 0.0065X_H,$$

$$H_4(\pi) \simeq 0.68 - 0.078i,$$

$$H_5(\pi) \simeq 0.2 - 0.001X_A + 0.004X_A^2$$

$$H_6(\pi) \simeq 0.68 - 0.078i - 0.007X_A + 0.014X_A^2,$$

$$H_7(\pi) \simeq 0.95 + 0.0004X_A - 0.0014X_A^2,$$

$$H_8(\pi) \simeq -0.068 + 0.08i + 0.002X_A - 0.0047X_A^2 - 0.009X_H,$$

$$H_9(\pi) \simeq -1.16 + 0.026i - 0.0015X_A - 0.0001X_A^2 - 0.003X_H,$$

$$H_{10}(\pi) \simeq -0.67 + 0.08i - 0.0096X_H,$$

$$H_{7\gamma}(\pi) \simeq 0.0004,$$

$$H_{8g}(\pi) \simeq -0.045. \tag{6.17}$$

The different sign between $\mathbf{C}_i$ and $\tilde{\mathbf{C}}_i$ appearing in Eq.(6.16) is due to the fact that $\langle K^0\pi^0|Q_i|B\rangle = -\langle K^0\pi^0|\tilde{Q}_i|B\rangle$, since the initial and the final states have different parity. Comparing the coefficients $H_i(\pi)$ with $H_i(\phi)$ and $H_i(\eta')$ in [5], one finds that the Wilson coefficients in these decay amplitudes are different. Thus it is naturally to have different CP asymmetries $S_{K^0\pi^0}$, $S_{K\phi}$ and $S_{K\eta'}$, unlike the SM prediction.

In order to understand the dominant SUSY contribution to the CP asymmetry $S_{K^0\pi^0}$, it is useful to present a numerical parametrization of the ratio of the amplitude R_π in terms of the relevant mass insertions. For the usual SUSY configurations that we have used in the previous sections, we obtain

$$R_\pi \simeq \left\{0.02 \times e^{-i0.4}\left(\delta_{LL}^d\right)_{23} - 40.4 \times e^{-i0.01}\left(\delta_{LR}^d\right)_{23}\right\} - \{L \leftrightarrow R\}$$
$$+ 0.15 \times e^{-i0.002}(\delta_{LL}^u)_{32} - 0.08 \times e^{-i0.013}(\delta_{RL}^u)_{32}. \tag{6.18}$$

From this result, it is clear that the largest SUSY effect is provided by the gluino contribution to the chromo-magnetic operator which is proportional to $\left(\delta_{LR}^d\right)_{23}$ and $\left(\delta_{RL}^d\right)_{23}$. For $\left(\delta_{LR}^d\right)_{23} \simeq 0.006 \times e^{i\pi/3}$ and all the other mass insertions set to zero, one finds $S_{K^0\pi^0} \simeq 0.34$ which coincides with the central value of the experimental results reported in Table 1. It is important to note that with such value of $\left(\delta_{LR}^d\right)_{23}$ the gluino contribution can account for the CP asymmetries $S_{K\phi}$ and $S_{K\eta'}$ as well [5]. Furthermore, if we consider the scenario where both chargino and gluino exchanges are contributed simultaneously, the result of R_π is enhanced and we can get smaller values of $S_{K^0\pi^0}$.

7. Conclusions

In this paper we have analyzed the supersymmetric contributions to the direct and mixing CP asymmetries and also to the branching ratios of the $B \to K\pi$ decays in a model independent way.

We have shown that, in the SM, the $R_c - R_n$ puzzle which reflects the discrepancy between the experimental measurements of the branching ratios and their expected results can not be resolved. Also the direct CP asymmetries $A^{CP}_{K^0\pi^-}$ and $A^{CP}_{K^0\pi^0}$ are very small while $A^{CP}_{K^0\pi^0}$ and $A^{CP}_{K^-\pi^+}$ are of the same order and can be larger. These correlations among the CP asymmetries are inconsistent with the recent measurements. Moreover the mixing CP asymmetry $S_{K^0\pi^0}$, which is expected to be $\sin 2\beta$, differs from the corresponding experimental data. The confirmation of these discrepancies will be a clear signal for new physics beyond the SM.

We have emphasized that the Z-penguin diagram with chargino in the loop and the chargino electromagnetic penguin can enhance the contribution of the electroweak penguin to $B \to K\pi$ which is supposed to play a crucial role in explaining the above mentioned discrepancies. We, however, found that these contributions alone are not enough to solve the $R_c - R_n$ puzzle. It turns out that a combination of gluino and chargino contributions is necessary to account for the results of R_c and R_n within the $b \to s\gamma$ constraints. Nevertheless, our numerical results confirmed that the general trend of SUSY models favors that the experimental result of R_c goes down.

We have also provided a systematic study of the SUSY contributions to the direct CP asymmetries for $B \to K\pi$ decays. We found that a large gluino contribution is essential to explain the recent experimental data. It is worth mentioning that a large gluino contribution is also important to accommodate another controversial results measured in the B factories, namely the mixing CP asymmetries $S_{\phi K}$ and $S_{\eta'K}$. Unlike the $R_c - R_n$ puzzle, we found that the CP asymmetries $A^{CP}_{K\pi}$ can be saturated by a single mass insertion $(\delta^d_{LR})_{23}$ contribution. It has been noticed that a large electroweak penguin is less favored by the CP asymmetries $A^{CP}_{K\pi}$. Therefore, one needs to optimize the gluino and chargino contributions in order to satisfy simultaneously the branching ratios and the CP asymmetries of $B \to K\pi$.

Finally we have considered the mixing CP asymmetry $S_{K^0\pi^0}$. We found, as in $S_{\phi K}$ and $S_{\eta'K}$, that the gluino contribution through the LR or RL mass insertion gives the largest contribution to $S_{K^0\pi^0}$. On the other hand, it is quite possible for the gluino exchanges to account for $S_{K^0\pi^0}$, $S_{\phi K}$ and $S_{\eta'K}$ at the same time.

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